On the Ambiguity of Job Search

Ying Tung Chan and Chi Man Yip*

Abstract

Who knows the underlying productivity distribution function? Interestingly, this ambiguous function is often referenced to make decisions including job creations, wage determinations, contract formulations, etc. To investigate how ambiguity shapes labour markets, we integrate ambiguity preferences into the Diamond-Mortensen-Pissarides (DMP) model. We find that ambiguity-averse job- and talent-hunters are conservative: they believe lower match-specific productivity levels to be more likely realized. This belief incents both sides to quickly accept a contract, shortening job search processes. Meanwhile, the belief reduces the valuation of future profits, depressing the creation of vacancies and lengthening unemployment durations. Our quantitative analysis indicates that but for the ambiguity, the American unemployment rate would have increased in the postwar era. This paper generalizes the DMP model, enhances our understanding of the labour market, and calls for policies concerning labour market information.

Keywords: Ambiguity Aversion; Unemployment; Skewness Shocks.

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1 Introduction

Outside option values are valuable information in labour markets. It is necessary to have such information to solve many important optimal stopping problems. For example, workers require such information to decide whether to accept a job offer by comparing the offer in hand with potential outside options. Firms make hiring decisions by comparing job candidates with alternatives. Although these values depend largely on the productivity distribution in the labour market, such a distribution function is largely unknown to workers and firms.

Prior literature assumes a common knowledge about the productivity distribution function (Rogerson et al., 2005). Workers and firms make decisions inside a model economists construct based on a specified productivity distribution function. Model misspecification may, therefore, arise: the underlying model workers and firms use to make decisions is potentially misspecified. Meanwhile, which alternative model (e.g., the distribution function) should be used to make decisions is ambiguous to both workers and firms. Yet, little is known about how an aversion to this ambiguity and fear about model misspecification affect the behaviors of workers and firms during job search processes.

To provide an in-depth study on the labour market effect of ambiguity, this paper purposes to (i) construct an analytically tractable search-theoretical model featuring ambiguity preferences, (ii) uncover major mechanisms through which ambiguity preferences affect two key labour market variables—the unemployment and wage rates, (iii) quantify unemployment attributable to the ambiguity, and (iv) discuss policy implications concerning labour market information.

Such an in-depth study requires a search-theoretical model featuring ambiguity preferences. Similar to other preferences such as risk- and loss-aversions, collecting the preference parameter of ambiguity aversion is empirically challenging.[1] Therefore, we uncover the parameter through the calibration of a theoretical model. However, the incorporation of ambiguity preferences into a search-theoretical framework is easier said than done. The Diamond-Mortensen-Pissarides (DMP) model is popular because of its comparative statics result that well describes the labour market. It can be easily extended to incorporate other labour market features partly because of its analytical tractability. This tractability relies heavily on the linearity of value functions to one another. Hence, the DMP model and its variants often assume agents to be risk- and ambiguity-neutral to avoid the nonlinearity of the value functions. We show that despite the nonlinearity, the DMP model featuring ambiguity aversion (Hansen and Sargent, 2008) preserves its analytical tractability and most of its intuitive comparative statics results.

Moreover, the analytical tractability allows us to clearly uncover major mechanisms through which the ambiguity affects important macroeconomic variables. Workers with stronger ambiguity aversion tend to believe that lower productivity levels are more likely to be realized. This belief reduces unemployed workers' outside option values and thus reservation wages; hence, they are more likely to accept job contracts from firms. As a result, the ambiguity speeds up the termination of the job search process, depressing unemployment.

[1]We acknowledge an experimental approach to uncover economic preference parameters (Borghans et al., 2009). Nevertheless, the micro-estimate from experiments may substantially differ from the macro-estimate from a calibrated model, which is more relevant to this paper.

Meanwhile, the lower reservation wage requires firms to compensate employees less, thereby reducing wages. The reduction in this compensation increases expected profits, encouraging the creation of vacancies. Hence, the ambiguity increases the contract acceptance rate and the supply of vacancies, both of which depress unemployment. Whereas conventional wisdom suggests that the revelation of labour market information helps job seekers get rid of unemployment, our result finds the opposite: the ambiguity towards the labour market helps the unemployed find jobs.

In contrast, ambiguity may affect firms' behaviors differently. Due to the ambiguity, firms are conservative: they believe lower productivity levels to be more likely realized. The belief reduces their outside option values, making them more likely to sign contracts with job applicants. Meanwhile, firms with stronger ambiguity aversion formulate a belief of lower expected profits, discouraging the creation of vacancies. While the increased acceptance rate of job applications depresses unemployment, the reduction in the supply of vacancies lengthens an average unemployment spell. The two compelling forces make it uncertain how a firm's ambiguity aversion affects unemployment.

We further study the role of ambiguity in the labour market through quantitative analyses. We find that the ambiguity increases employment in the United States. If both workers and firms became ambiguity-neutral in reality, unemployment would have increased, especially in a recession. Our quantitative analyses find that the productivity distribution tends to have a thicker left-tail in downturns, consistent with the skewness shocks documented in Salgado et al. (2019). We find that these skewness shocks catalyze the employment effect of ambiguity in downturns. As a result, the ambiguity depresses unemployment more in slumps than in booms. But for the ambiguity, the American unemployment rate would be much larger in recession years.

An interesting policy dilemma concerning labour market information emerges. According to our quantitative analysis, the revelation of labour market information to workers increases unemployment, and the effect is substantial. Therefore, passing labour market information to workers hurts an economy in slumps by worsening unemployment. In contrast, the removal of firms' ambiguity depresses unemployment, but the impact is small. Quite often, policymakers may want to encourage the creation of job vacancies by providing firms with abundant labour market information. Our findings suggest that it helps, but they are simply *busy doing nothing*: the policy is ineffective.

This paper builds on insights from two sets of literature. It incorporates the Hansen and Sargent (2008) type of ambiguity preferences into a search-theoretical model. The DMP model is chosen partly because its analytical tractability allows the model to be generally and easily extended to incorporate other labour market features including on-the-job searches (Dolado et al., 2009; Postel-Vinay and Turon, 2014), firing costs (Postel-Vinay and Turon, 2014; Vindigni et al., 2014), job referral (Calvó-Armengol and Zenou, 2005; Galenianos, 2014), heterogeneity in sectors (Acemoglu, 2001; Albrecht et al., 2018), discrimination (Sasaki, 1999; Rosén, 2003), human capital accumulation (Cairo and Cajner, 2018), etc. Our model generalizes the DMP model by allowing agents to be ambiguity-averse without losing its analytical tractability. A similar generalization can therefore be applied to this broad literature.

This paper's focus on the ambiguity in the labour market fits broadly within the application of robust control methods (Cao et al., 2005; Adam and Woodford, 2012; Athanassoglou and Xepapadeas, 2012;

Croce et al., 2012; Djeutem and Kasa, 2013; Ilut and Schneider, 2014). Nishimura and Ozaki (2004) incorporates Knightian uncertainty into a job-seeker problem, showing that Knightian uncertainty reduces the reservation wage. In addition to the worker's problem, this paper studies vacancies' problems, completing the search process between workers and firms under ambiguity. Moreover, this paper quantifies unemployment attributable to workers' and firms' ambiguity and examines the relationship between ambiguity and unemployment over business cycles.

This paper also complements a series of influential papers that study the relationship between unemployment and the productivity distribution (Bloom et al., 2007; Bloom, 2009; Schaal, 2017; Bloom et al., 2018; Salgado et al., 2019). Schaal (2017) shows that unemployment fluctuations could be explained in large part by the volatility of productivity. Salgado et al. (2019) finds that the skewness shock depresses employment growth in recessions. This paper contributes to this literature by linking the ambiguity towards a productivity distribution to the skewness of the productivity distribution, enhancing our understanding of the labour market over business cycles. This linkage, though important in understanding unemployment fluctuations, has not been addressed in previous formal models.

This paper is organized as follows. Section 2 presents a basic model setting. Section 3 characterizes a steady-state equilibrium and explores the labour market effects of ambiguity preferences. Comparative statics are analytically shown in Section 4. Section 5 quantifies the labour market effects of ambiguity. Section 6 concludes this paper.

2 The Basic Model

This section constructs a search-theoretical model that allows both workers and vacancies to be ambiguity-averse.[2] In contrast to the literature, we do not assume that the data-generating process of an economy is known to agents—workers and firms. To make decisions, the agents utilize an approximating model that best describes the data-generating process. Meanwhile, they fear the possible misspecification of this approximating model. Hence, decisions are not made simply according to the rational expectation paradigm. Instead, the agents incorporate a penalty function into their value functions to distort the approximating model so that the fear of model misspecification can be considered. In addition to the decisions on job searches and vacancy creations as in the conventional search and matching model, the model of choice is chosen so that the decisions during search processes are made to maximize the value function under the worst-case scenario. Following the convention in the literature on ambiguity preferences, we call the optimal model, which is chosen by an ambiguity-averse agent, a distorted model.

Consider a discrete-time economy with a fixed labour force, without loss of generality normalized to unity. Each worker has an infinite horizon and is either employed or unemployed.[3] A vacancy is

[2]To shed light on the labour market effects of ambiguity preferences, a search and matching model is constructed at its simplest. Readers who are interested in the model with other elements such as learning are referred to Moscarini (2005), Gonzalez and Shi (2010), and Papageorgiou (2014) for a search and matching model with learning and Epstein and Schneider (2008) for the ambiguity preferences with learning.

[3]The model assumes no decision on labour supply, either the number of working hours or labour force participation. This simplification is standard (Mortensen and Pissarides, 1994; Shimer, 2005; Hall, 2005; Hagedorn and Manovskii, 2008; Hall and

either filled or unfilled with a worker. The measure of the vacancy is endogenized. Both workers and vacancies are ambiguity-averse, know the degree of ambiguity aversion of each other, and share an identical discounted factor $\beta \equiv 1/(1+r)$, where r is a real interest rate.

An unemployed worker receives unemployment benefit b and meets an unfilled vacancy via a matching technology $M(u,v)$, where u and v are the numbers of unemployed workers and unfilled vacancies, respectively. The matching technology $M(u,v)$ gives the number of pairwise meetings per period. Assume that $M(u,v)$ is homogeneous of degree one and increasing in each argument so that $M(u,v)/v = M(u/v,1)$. We denote the rate at which an unfilled vacancy meets an unemployed worker $q(\theta) \equiv M(u,v)/v$, where $\theta \equiv v/u$ is the market tightness. It follows that $q(\theta)$ is a decreasing function and that the rate at which an unemployed worker meets an unfilled vacancy $M(u,v)/u = \theta q(\theta)$ is an increasing function of θ, as in Pissarides (2000). We also make standard Inada-type assumptions on $M(u,v)$ so that $\lim_{\theta\to 0} q(\theta) = \infty$, $\lim_{\theta\to\infty} q(\theta) = 0$, $\lim_{\theta\to 0} \theta q(\theta) = 0$, and $\lim_{\theta\to\infty} \theta q(\theta) = \infty$.

When an unemployed worker and an unfilled vacancy meet, a match-specific productivity level δ is realized. We assume δ follows a cumulative distribution function $F(\delta)$ over $\mathbb{R}_+$, and the corresponding probability density function is denoted by $f(\delta)$. Given δ, they bargain on the wage. The unemployed worker will reject the job offer if the expected lifetime utility that would result from remaining in the status of an unemployed worker exceeds that of employment with productivity δ.

In contrast to the literature, agents are not confident that $F(\delta)$ is the "true" distribution. As shown in Hansen and Sargent (2008), an ambiguity-averse agent will replace the density function $f(\delta)$ with an alternative density $\hat{f}(\delta)$ to account for his fear of losses arising from model misspecification. Following Hansen and Sargent (2008), this class of problem can be formulated as choosing the likelihood ratio $m(\delta) \equiv \hat{f}(\delta)/f(\delta)$ for all δ to optimize a value function that is a function of an ambiguity preference $\alpha \leq 0$ and a measure of the distance between the two distributions. We follow Hansen and Sargent (2008) to use $\mathbb{E}_\delta m(\delta)\ln m(\delta)$, known as relative entropy between $f(\delta)$ and $\hat{f}(\delta)$, to measure the Kullback Leibler distance between two distributions. In this model, this relative entropy measures the discrepancy between the approximating and distorted models. Denote $J^E(\delta)$ and J^U as the value functions of employment and unemployment, respectively. The value function of an unemployed worker J^U must satisfy the following function:

$$
\begin{aligned}
J^U \quad &= \quad \min_{m(x)} b + \beta \mathbb{E}_x \left[\theta q(\theta) \left(m(x) \max\{J^E(x), J^U\} - \frac{1}{\alpha} m(x) \ln m(x) \right) + (1-\theta q(\theta)) J^U \right] \\
\text{subject to} \quad & \int m(x) dF(x) = 1. \qquad (1)
\end{aligned}
$$

An unemployed worker chooses the likelihood ratio $m(\delta)$ to minimize the value function, and the decisions during a job search process are made to maximize this minimized value function. We can also interpret the choice of $m(\delta)$ as follows. Given the approximating model $F(\delta)$, an unemployed worker chooses the distorted model to minimize the value function. The decisions are then made to maximize

Milgrom, 2008; Fujita and Ramey, 2012; Michaillat, 2012), and is in line with empirical regularities: cyclical variations in total working hours (unemployment) basically arise from changes in the amount of employment but not changes in working hours per worker (labour force participation) (Shimer, 2010).

the value function under the worst-case scenario in which the probability density function is $\hat{f}(\delta)$.

$\alpha \leq 0$ is a penalty parameter for relative entropy, which captures the degree of a worker's ambiguity aversion. When $\alpha = 0$, workers are said to be ambiguity-neutral. The more workers fear model misspecification, the lower the α is.

The optimal likelihood ratio is given by

$$\frac{\hat{f}(\delta)}{f(\delta)} = \frac{e^{\alpha \max\{J^E(\delta), J^U\}}}{\int_0^{\infty} e^{\alpha \max\{J^E(x), J^U\}} dF(x)}. \quad (2)$$

[4]

We will show that the partial derivative $\partial J^E(\delta)/\partial\delta$ is positive. Hence, the optimal likelihood ratio decreases with δ. Intuitively, an ambiguity-averse unemployed worker will choose the "distorted distribution" in which a higher probability is assigned to a lower match-specific productivity level. In the limiting case where α approaches zero from below, equation (2) implies that $\hat{f}(\delta) = f(\delta)$. In this case, the value function of unemployment becomes

$$J^U = b + \beta \mathbb{E}_x \Bigg(\theta q(\theta) \max\{J^E(x), J^U\} + (1 - \theta q(\theta)) J^U \Bigg),$$

which is the case in the absence of the fear of model misspecification. When a worker does not worry about model misspecification, his belief of the probability density function is identical to the approximating one. That is, the value function of unemployment in the conventional search and matching model is a special case of ours when the entropy penalty parameter α equals zero.

An employed worker with a match-specific productivity level δ receives a bargained wage $w(\delta)$ and, at the end of each period, faces a separation shock at a rate of λ. When the shock arrives, the worker becomes unemployed. The discounted present values of employment $J^E(\delta)$ can be written as follows:

$$J^E(\delta) = w(\delta) + \beta \Bigg(\lambda J^U + (1 - \lambda) J^E(\delta) \Bigg). \quad (3)$$

A filled vacancy generates a production value δ, pays a worker $w(\delta)$, and faces a separation shock at a rate of λ. When the shock arrives, a filled vacancy becomes unfilled. Denote $J^F(\delta)$ and J^V as asset values of a filled and an unfilled vacancy, respectively. The asset value of a filled vacancy $J^F(\delta)$ can be written as follows:

$$J^F(\delta) = \delta - w(\delta) + \beta \Bigg(\lambda J^V + (1 - \lambda) J^F(\delta) \Bigg). \quad (4)$$

An unfilled vacancy pays maintenance cost $c > 0$ and faces a probability $q(\theta)$ of being filled. When an unfilled vacancy meets an unemployed worker, it will agree to form a match with the worker if the match-specific productivity level is high enough such that $J^F(\delta) \geq J^V$. An unfilled vacancy, being ambiguity-averse, maximizes the minimum expected outcome. To do so, a vacancy chooses $m_v(\delta) \equiv \hat{f}_v(\delta)/f(\delta)$, a likelihood ratio, to maximize the minimum expected outcome, where $\hat{f}_v(\delta)$ is a probability density function of the distorted model chosen by an unfilled vacancy. Hence, the asset value of an unfilled vacancy J^V can be written as

$$J^V = \min_{m_v(x)} -c + \beta \mathbb{E}_x \left[q(\theta) \left(m_v(x) \max\{J^F(x), J^V\} - \frac{1}{\alpha_v} m_v(x) \ln m_v(x) \right) + (1 - q(\theta)) J^V \right]$$

subject to $$\int m_v(x) dF(x) = 1, \tag{5}$$

where $\alpha_v \leq 0$ is anther entropy penalty parameter, capturing the degree of ambiguity aversion of a vacancy. This problem can be solved in a similar way as the minimization problem facing unemployed workers (2). The corresponding likelihood ratio is given by

$$\frac{\hat{f}_v(\delta)}{f(\delta)} = \frac{e^{\alpha_v \max\{J^F(\delta), J^V\}}}{\int_0^\infty e^{\alpha_v \max\{J^F(x), J^V\}} dF(x)}. \tag{6}$$

The ambiguity preferences of workers and vacancies share the same notion in the optimal likelihood ratio: more ambiguity-averse agents tend to believe that a lower productivity level is more likely to be realized. When an unemployed worker and an unfilled worker meet, a match-specific productivity level δ is realized, and they bargain on the wage to maximize the generalized Nash product as follows:

$$w(\delta) \equiv \arg\max (J^E(\delta) - J^U)^\eta (J^F(\delta) - J^V)^{1-\eta},$$

where $\eta \in (0, 1)$ is the bargaining power of workers.[5] Understanding the ambiguity preferences is important in wage determination because the degree of ambiguity aversion affects the bargained wage through the outside option values of workers and vacancies (i.e., J^U and J^V). For simplicity, we assume that the ambiguity preferences of workers and vacancies are common information.[6] Simple algebra gives the following sharing rule:

$$J^E(\delta) - J^U = \eta \Big(J^E(\delta) - J^U + J^F(\delta) - J^V \Big). \tag{7}$$

Intuitively, a match surplus of a worker is a fraction η of the total match surplus. The assumption of free entry and exit is made; hence, rent is exhausted, and thus

$$J^V = 0. \tag{8}$$

Denote δ_u^R as the reservation productivity level, below which an unemployed worker will not accept any job offer. Since an unemployed worker will accept the job offer only if $J^E(\delta) \geq J^U$, $J^E(\delta_u^R) = J^U$.

[5] Maximizing this generalized Nash product is only one of the wage determination methods. So long as the wage determination method returns an employed worker a fraction of total matching rent, the ambiguity aversion of a worker can affect labour market outcomes. Therefore, the implications of this model are not driven by the proposed wage determination mechanism.

[6] In reality, interviews enable vacancies to understand workers' match-specific productivity levels and attitudes toward ambiguity. For example, an expected salary on a resume could reveal an unemployed worker's belief of his/her expected productivity and/or ambiguity preference. Another example is an aptitude test, which could reveal the ambiguity preferences of workers. Readers who are interested in the model in which agents have asymmetric information on the degree of ambiguity aversion are referred to Ahn (2007).

A reservation productivity threshold for an unfilled vacancy δ_v^R is defined in a similar way; therefore, $J^F(\delta_v^R) = J^V$. Using the sharing rule (7), one can easily show that the reservation productivity levels for a worker and a vacancy are identical. Hereafter, denote the reservation productivity level as δ^R. Since both $J^E(\delta)$ and $J^F(\delta)$ strictly increase with δ, the δ^R is unique. Hence, when unemployed workers and unfilled vacancies meet, they accept the offer for all $\delta \geq \delta^R$. Using equations (3), (4), (7), and (8), the wage equation is derived as follows:

$$w(\delta) = \eta\delta + (1-\eta)\beta r J^U. \tag{9}$$

Hence, a wage is a fraction of production value plus a fraction of the worker's outside option value. Using equations (3) and (9), a reservation wage is given by

$$w(\delta^R) = \delta^R = \beta r J^U. \tag{10}$$

Hence, the reservation wage $w(\delta^R)$, the reservation productivity level δ^R, and the worker's outside option value are identical. If the realized productivity level δ exceeds δ^R, the bargained wage is given by

$$w(\delta) = (1-\eta)\delta^R + \eta\delta. \tag{11}$$

A worker is compensated with a fraction of the reservation productivity level δ^R plus a fraction of the production value.

A steady-state unemployment rate is determined by equating the outflow and inflow of unemployment and is given by

$$u = \frac{\lambda}{\lambda + [1 - F(\delta^R)]\theta q(\theta)}. \tag{12}$$

A steady-state unemployment rate strictly increases with the reservation productivity level δ^R because a higher δ^R reduces a job offer acceptance rate and thus lengthens an unemployment spell.

3 Characterization of a Steady-State Equilibrium

This section characterizes a steady-state equilibrium and explores the labour market effects of ambiguity preferences. We show that our generalization of the DMP model is highly analytically tractable. The tractability allows us to clearly uncover the mechanisms through which ambiguity preferences affect the labour market.

Definition 1. *A steady-state equilibrium is defined as* $\{\hat{f}(\delta),\ \hat{f}_v(\delta),\ \delta^R,\ w(\delta),\ u,\ \theta,\ J^E(\delta),\ J^U,\ J^F(\delta),\ J^V\}$ *such that equations (1), (3) to (5), (7), (8), (10), and (12) are satisfied for all* $\delta \geq \delta^R$*, and equations (2) and (6) are satisfied for all* $\delta \in \mathbb{R}_+$.

Using equations (1) and (2), the value function of an unemployed worker is written by

$$\beta r J^U = b + \frac{\beta\theta q(\theta)}{\alpha} \ln\left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\alpha(J^E(x)-J^U)} dF(x)\right). \tag{13}$$

A rise in market tightness improves the chances of meeting unfilled vacancies, enhancing the outside option value of unemployed workers. Using equations (5) and (6), the value function of an unfilled vacancy is given by

$$\beta r J^V = -c + \frac{\beta q(\theta)}{\alpha_v} \ln\left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\alpha_v(J^F(x)-J^V)} dF(x)\right). \tag{14}$$

A rise in market tightness makes it more competitive to meet unemployed workers, deteriorating the outside option value of vacancies. When $\alpha = 0$ and $\alpha_v = 0$, the preceding equations reduce to

$$\begin{aligned}\beta r J^U &= b + \beta\theta q(\theta)\int_{\delta^R}^{\infty}(J^E(x) - J^U)dF(x) \text{ and}\\ \beta r J^V &= -c + \beta q(\theta)\int_{\delta^R}^{\infty}(J^F(x) - J^V)dF(x).\end{aligned}$$

They are the value functions of unemployed workers and unfilled vacancies in the model in which workers and vacancies are ambiguity-neutral.[7] Thus, the fundamental setup in the literature on the search equilibrium model is a special case of the present model, when both α and α_v equal zero. Interestingly, the four value functions (i.e., $J^E(\delta)$, $J^F(\delta)$, J^U, and J^V) preserve linearity only if both workers and vacancies are ambiguity-neutral (i.e., α and α_v are zero), making the DMP model analytically tractable. According to equations (13) and (14), the two outside option values are generally nonlinear: J^U is a nonlinear function of J^E and J^V is a nonlinear function of J^F. We will show that despite the nonlinearity, our generalization of the DMP model preserves its analytical tractability.

Substituting equations (2), (3), (9), and (10) into equation (13), simple algebra gives

$$\delta^R = b + \frac{\beta\theta q(\theta)}{\alpha} \ln\left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)} dF(x)\right). \tag{15}$$

Similarly, substituting equations (4), (6), (9), and (10) into equation (14) yields

$$c = \frac{\beta q(\theta)}{\alpha_v} \ln\left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha_v(1-\eta)(1+r)}{r+\lambda}(x-\delta^R)} dF(x)\right). \tag{16}$$

The derivations of equations (15) and (16) are shown in Appendix 7.2. The steady-state equilibrium is characterized by the intersection of loci (15) and (16) as shown in Figure 1. Locus (15) equates the outside option value of unemployed workers to the reservation productivity level. A rise in market tightness increases the chance of meeting vacancies, causing the outside optional value of unemployed workers to increase. A higher reservation productivity level is, therefore, required for an unemployed

[7] When $\alpha = 0$ and $\alpha_v = 0$, $F(\delta^R) + \int_{\delta^R}^{\infty} e^{\alpha(J^E(x)-J^U)}dF(x) = 1$ and $F(\delta^R) + \int_{\delta^R}^{\infty} e^{\alpha_v(J^F(x)-J^V)}dF(x) = 1$. Applying L'Hopitals' Rule to equations (13) and (14) yields the result.

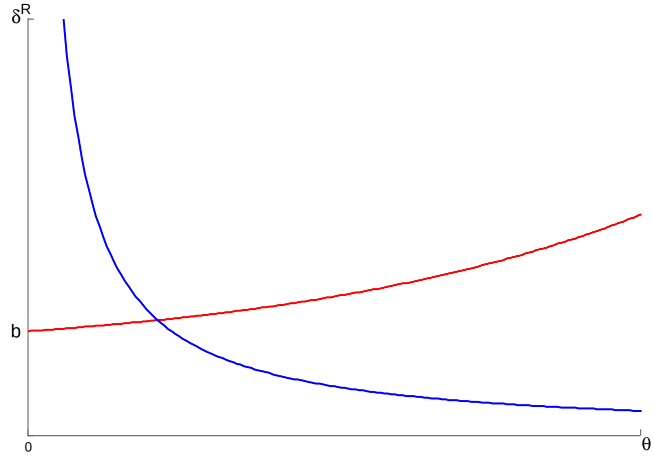


Figure 1: Equilibrium Determination

worker to accept any job offer.[8] When θ goes to zero, δ^R approaches b.

The locus (16), along which an unfilled vacancy makes zero profit, slopes downward. A rise in market tightness reduces the probability that vacancies will be filled, thereby lowering J^V. Hence, a reservation productivity level falls so as to raise the transition rate of being filled to maintain zero profit. When θ tends to zero, δ^R approaches infinity. By the continuity of the two functions, the intermediate value theorem ensures that the two loci intersect. Since both loci are strictly monotone, they only intersect once at $\delta^{R*} \in (b, \infty)$ and $\theta^* \in (0, \infty)$. The following proposition summarizes the findings.

Proposition 1. *For all $\alpha \leq 0$ and $\alpha_v \leq 0$, there exists a unique steady-state equilibrium, which is characterized by equations (15) and (16). In the equilibrium, $\delta^{R*} \in (b, \infty)$ and $\theta^* \in (0, \infty)$.*

While the literature shows the existence of the steady-state equilibrium in the absence of ambiguity, Proposition 1 generalizes the result: it shows the existence of a unique steady-state equilibrium in the model for all ambiguity preferences $\alpha \leq 0$ and $\alpha_v \leq 0$.

Proposition 2. *When workers become more ambiguity-averse, θ^* increases, δ^{R*} decreases, $w^*(\delta)$ decreases for all $\delta \geq \delta^{R*}$, and u^* decreases. When firms become more ambiguity-averse, θ^* decreases, δ^{R*} decreases, and $w^*(\delta)$ decreases for all $\delta \geq \delta^{R*}$.*

Proof. See Appendix 7.3. □

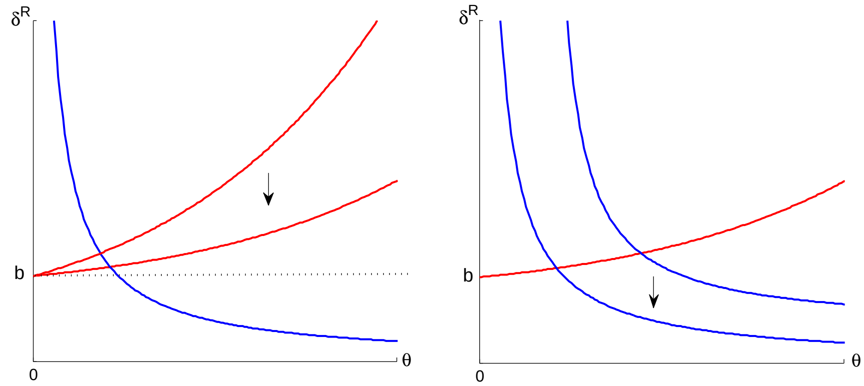
Figure 2 demonstrates the labour market effects when workers and firms become more ambiguity-averse. When workers become more ambiguity-averse, the locus (15) rotates clockwise and the zero-

[8]Denote $\Upsilon(\delta^R) \equiv F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)} dF(x) \leq 1$. Applying the Leibniz integral rule to differentiate $\ln \Upsilon(\delta^R)$ with respective to δ^R, we have

$$\frac{\partial \ln \Upsilon(\delta^R)}{\partial \delta^R} = -\frac{1}{\Upsilon(\delta^R)} \frac{\alpha\eta(1+r)}{r+\lambda} \int_{\delta^R}^{\infty} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)} dF(x) > 0.$$

Hence, δ^R and θ are positively associated in the locus (15). Following a similar procedure, one could verify that δ^R and θ are negatively associated in the locus (16).

Figure 2: The Effect of Stronger Ambiguity Aversion on the Reservation Productivity Threshold



Notes: The left (right) panel shows the effect of stronger ambiguity aversion of workers (vacancies).

profit condition (16) is unaffected. A new steady-state equilibrium occurs at the intersection of two loci, with a lower value of δ^{R*} and a larger value of θ^* than the old ones.

Intuitively, ambiguity-averse workers do not simply make decisions based on the approximating model. With the fear of model specification, these workers make decisions based on the distorted model in which a higher likelihood is assigned to a lower match-specific productivity level. Since the match-specific productivity level is lower in the distorted model than the approximating one, their outside option values become smaller. As a result, it requires firms to pay lower wages to compensate workers with stronger ambiguity aversion. When unemployed workers and unfilled vacancies meet, a lower match-specific productivity level is sufficient to pay the lower wage, causing the reservation productivity levels to decline. With a lower δ^{*R}, unemployed workers are more likely to accept job contracts. Hence, the ambiguity deteriorates an average match quantity, providing a rationale to our traditional thought: the revelation of labour market information improves job qualities.

Meanwhile, the reduction in wages increases flow profits. Thus, the supply of vacancies increases until the creation of vacancies exhausts the rent in the equilibrium. With more vacancies, unemployed workers become more likely to meet unfilled vacancies. In a steady-state equilibrium, the increases in the contract acceptance rate and the supply of vacancies lower the unemployment rate. In other words, if workers became ambiguity-neutral in reality, the unemployment rate would have been larger. Whereas conventional wisdom suggests that the revelation of labour market information to unemployed workers helps them get rid of unemployment, our result finds the opposite: the ambiguity indeed reduces unemployment. We will demonstrate the empirical significance of the unemployment effect in Section 5.

Similarly, vacancies with stronger ambiguity aversion make decisions based on a distorted model with a lower average match-specific productivity level. Hence, they expect profits to be lower, thereby reducing their outside option values. As a result, they are willing to formulate a production unit with workers even though a lower match-specific productivity level is realized. In other words, the reservation productivity level decreases. Meanwhile, the reduction in the expected profits discourages the supply of vacancies. While firms become more likely to sign contracts with workers due to the decrease in the reservation productivity level, the reduction in the supply of vacancies lowers the likelihood that unemployed workers meet unfilled vacancies. Therefore, it is uncertain to conclude whether the stronger ambiguity aversion of vacancies will increase or decrease unemployment. This result can also be seen by totally differentiating a steady-state unemployment rate u^* with respect to α_v as follows:

$$\frac{du^*}{d\alpha_v} = \underbrace{\frac{\partial u^*}{\partial \delta^{R*}}}_{+} \underbrace{\frac{d\delta^{R*}}{d\alpha_v}}_{+} + \underbrace{\frac{\partial u^*}{\partial \theta^*}}_{-} \underbrace{\frac{d\theta^*}{d\alpha_v}}_{+}.$$

When firms become more ambiguity-averse, α_v is smaller. Due to the stronger ambiguity preference, unemployment decreases through the reduction in the reservation productivity level but increases through the increase in the supply of vacancies. Because of this indetermination, we will conduct a quantitative analysis on the unemployment effect in Section 5.

Importantly, the underlying mechanisms, through which ambiguity preferences affect the labour market, are robust to other wage-setting methods. This paper follows the literature and uses bargained wages that maximize generalized Nash products. Consider an extreme case in which wages are completely independent of the worker's reservation wage and are exogenously given. Given these exogenous wage levels, vacancies with higher degrees of ambiguity aversion believe that lower productivity levels are more likely to be realized. This belief shrinks their expected profit and thus lowers their reservation productivity threshold, reducing the supply of vacancies. Therefore, ambiguity aversion does affect other labour market variables even though wages are determined not by maximizing the generalized Nash products.

Next, we explore the limiting properties of α and α_v. According to the optimal likelihood ratio (2), for all $\delta > \delta^{R*}$, we have

$$\begin{aligned}
\frac{\hat{f}(\delta)}{f(\delta)} &= \frac{e^{\alpha J^E(\delta)}}{\int_0^{\delta^{R*}} e^{\alpha J^U} dF(x) + \int_{\delta^{R*}}^{\infty} e^{\alpha J^E(x)} dF(x)} \\
&= \frac{1}{\int_0^{\delta^{R*}} e^{\alpha(J^U - J^E(\delta))} dF(x) + \int_{\delta^{R*}}^{\infty} e^{\alpha(J^E(x) - J^E(\delta))} dF(x)}.
\end{aligned}$$

When α approaches negative infinity, the R.H.S. goes to zero because $J^U - J^E(\delta) < 0$ for all $\delta > \delta^{R*}$. Therefore, the optimal likelihood ratio $\hat{f}(\delta)$ is zero for all $\delta > \delta^{R*}$ and is $f(\delta)/F(\delta^{R*})$ for all $\delta \in [0, \delta^{R*}]$. In other words, if workers are sufficiently ambiguity-averse, they will choose to believe that it is impossible to have any match-specific productivity exceeding the reservation level. If this is the case, the outside option value of workers leaves with unemployment benefit b. Hence, δ^{R*} will approach b: the locus (15) becomes a horizontal line $\delta^{R*} = b$, as shown in Figure 2. Equivalently, one can show that when α approaches negative infinity, the second term in equation (15) vanishes, and hence, $\delta^{R*} = b$. The labour market persists: unemployed workers accept all the job offers as long as the match-specific productivity exceeds b.

Interestingly, the labour market collapses when vacancies are sufficiently ambiguity-averse. Applying a similar argument to the optimal likelihood ratio (6), one can show that when α_v approaches negative infinity, the optimal likelihood ratio $\hat{f}_v(\delta)$ is zero for all $\delta > \delta^{R*}$ and is $f_v(\delta)/F(\delta^{R*})$ for all $\delta \in [0, \delta^{R*}]$. In other words, vacancies tend to believe that the likelihood of match-specific productivity exceeding the reservation level is zero, driving the supply of vacancies and thus market tightness to zero. Equivalently, one can show that when α_v approaches negative infinity, the R.H.S. in equation (16) tends to zero. To maintain the zero-profit condition (16), θ^* goes to zero. In this case, the labour market collapses.

Proposition 3. *If $\alpha \to -\infty$, the labour market persists: the reservation productivity level δ^{R*} equals unemployment benefit b. If $\alpha_v \to -\infty$, the labour market collapses: there exists no vacancies.*

Before closing this section, it is important to highlight that job search behaviors are not observationally equivalent under ambiguity aversion and risk aversion. For example, this section shows that a higher degree of workers' ambiguity aversion reduces unemployment, we show in Online Appendix B that the unemployment effect of a higher degree of risk aversion is indeterminate.

4 Comparative Statics

The DMP model can be easily extended to incorporate other labour market features partly because of its analytical tractability and partly because of its comparative statics results that well describe the labour market. While the previous section shows that our generalization of the DMP model is highly analytically tractable, we are yet to know whether our generalization preserves those comparative statics results. In particular, this section investigates how production technology, maintenance costs, unemployment benefits, and matching technology affect other labour market outcomes in our model.

Advances in Production Technology. We first explore the impacts of advances in production technology on the labour market. Consider a permanent productivity shock to the economy so that a productivity distribution function F is transformed into a new one G, where G first-order stochastically dominates F (i.e., $G \succeq_{FOSD} F$).

With the advance in productivity technology, the average match-specific productivity level is higher. Unemployed workers are willing to wait longer for a higher realized productivity level, causing the reservation productivity level to rise. In other words, the outside option value of unemployed workers increases. It is clear from the wage equation (11) that filled vacancies are required to pay more to compensate for the increased outside option value. Therefore, wage levels climb up for all δ exceeding the reservation level.

The impact of the increase in productivity on market tightness is indeterminate. Such an improvement increases the average match-specific productivity level and thus the expected profits. This primary effect induces more supplies of vacancies and thus increases market tightness. Meanwhile, as explained above, such an improvement increases the reservation productivity level, making it harder for unemployed workers to accept a job offer. This general equilibrium effect depresses the supply of vacancies and thus lowers market tightness. Since we are uncertain about the magnitude of the two forces, the total effect of the advance in production technology on market tightness is indeterminate. As market tightness is one of the key variables in determining a steady-state unemployment rate, the effect on an unemployment rate is also indeterminate. The following proposition summarizes our analytical findings.

Proposition 4. *If F and G are two productivity distributions where $G \succeq_{FOSD} F$, $w_G(\delta) > w_F(\delta)$ for all $\delta \geq \delta_G^R$.*

Proof. See Appendix 7.4. □

Increases in Unemployment Benefit and Vacancy Maintenance Cost. We explore the impact of increased unemployment benefits on the labour market. A rise in unemployment benefits increases the outside option value of the unemployed worker. As a result, unemployed workers become more "picky" and require a higher realized productivity level to accept a job offer, thereby increasing the reservation productivity level δ^{R*}. With the increased outside option value, filled vacancies are required to pay higher wage levels to employed workers for all $\delta \geq \delta^{R*}$, depressing profits. With lower expected profits, the zero-profits condition drives some unfilled vacancies out of the labour market in a steady-state equilibrium. Consequently, the supplies of vacancies and thus market tightness θ^* drop. While the rise in δ^{R*} increases the job acceptance rate, the fall in θ^* reduces the likelihood that unemployed workers meet unfilled vacancies. These two effects lead to a higher unemployment rate in a steady-state equilibrium.

Next, we examine the effects of increased vacancy maintenance cost on the labour market. The increased vacancy maintenance cost makes it more costly to create an unfilled vacancy. In other words, a rise in this maintenance cost requires higher expected flow profits to maintain zero profit, discouraging unfilled vacancies to stay in the labour market. Hence, market tightness drops. The decrease in the supplies of unfilled vacancies makes it harder for unemployed workers to meet unfilled vacancies. As a result, the outside option value of workers declines, causing the reservation productivity level and thus wages to fall. Totally differentiating a steady-state unemployment rate with respect to c, we have

$$\frac{du^*}{dc} = \underbrace{\frac{\partial u^*}{\partial \delta^{R*}}}_{+} \underbrace{\frac{d\delta^{R*}}{dc}}_{-} + \underbrace{\frac{\partial u^*}{\partial \theta^*}}_{-} \underbrace{\frac{d\theta^*}{dc}}_{-}.$$

While the fall in θ^* reduces the likelihood that unemployed workers meet vacancies, the lower δ^{R*} makes unemployed workers less picky to job offers. Consequently, the impact of increased maintenance cost on unemployment is indeterminate. Proposition 5 summarizes the findings.

Proposition 5. *A rise in unemployment benefit increases wage levels $w(\delta)$ for all $\delta \geq \delta^{R*}$ and unemployment. A rise in maintenance cost reduces θ^*, δ^{R*}, and $w(\delta)$ for all $\delta \geq \delta^{R*}$.*

Proof. See Appendix 7.5. □

Advances in Matching Technology. An improvement in matching technology increases the matching rate between unemployed workers and unfilled vacancies. A higher matching rate allows both of them to re-search another offer more easily if either side rejects a current offer. Consequently, they become more "picky" during the searching process, driving up the reservation productivity level. Again, a natural consequence is that the wage $w(\delta)$ increases for $\delta \geq \delta^{R*}$ in response to the increase in δ^{R*}. However, the impact on θ^* is uncertain. On one hand, the rise in a matching rate increases the outside option value of vacancies, thereby providing incentives to create vacancies. On the other hand, the rise in wages reduces expected profits, depressing the supply of vacancies. Hence, we are uncertain about the effect of the advances in matching technology on θ^* and thus a steady-state unemployment rate. Interestingly, the advances in matching technology do not necessarily imply a lower unemployment rate in a steady-state equilibrium. The following proposition summarizes our findings.

Proposition 6. *If $q_1(\theta) > q_2(\theta)$ for all $\theta \in \mathbb{R}_{++}$, $\delta_1^{R*} > \delta_2^{R*}$ and thus $w_1(\delta) > w_2(\delta)$ for all $\delta \geq \delta_1^{R*}$ in the equilibrium.*

Proof. See Appendix 7.6. □

We have shown analytically how variations in production technology, unemployment benefits, maintenance costs, and matching technology affect the labour market. While our model is shown in Section 3 to nest the canonical one with ambiguity neutrality (i.e., $\alpha = 0$ and $\alpha_v = 0$), this section generalizes the comparative statics results so that all the propositions in this section hold for all $\alpha \leq 0$ and $\alpha_v \leq 0$.

5 Quantitative Analysis

This section quantifies the extent to which ambiguity towards a productivity distribution function affects labour market activities. Such quantitative analysis is informative: while section 3 uncovers the major mechanisms through which ambiguity towards a productivity distribution function affects job-seeking behaviors, this section measures the empirical significance of the impacts.

This calibration exercise can be divided into three sections. We will calibrate our model to the economy of the United States in the postwar era. Section 5.1 will calibrate most parameters, followed by the calibration of entropy penalty parameters α and α_v in section 5.2. Section 5.3 will quantify the extent to which ambiguity affects labour market activities. It will examine the impact of ambiguity over business cycles and discuss policy implications.

5.1 Calibrating DMP Model under Ambiguity Neutrality

This section calibrates parameters to match our model to the U.S. postwar economy (i.e., 1948-2007). In this calibration exercise, we assume agents to be ambiguity-neutral so that entropy penalty parameters are zero (i.e., $\alpha = \alpha_v = 0$). We will calibrate the two entropy penalty parameters in the next section.

Each model period is assumed to last a quarter. We follow Shimer (2005) to set a quarterly interest rate to be 0.012.

We follow Michaillat (2012) to specify a matching function as $M(u, v) = au^{\gamma}v^{1-\gamma}$. Not only does the specification cohere with the empirical regularity documented in Petrongolo and Pissarides (2001) but it also ensures the elasticity of $q(\theta)$ with respect to θ to be a constant. As such, we can calibrate the elasticity γ to a worker's bargaining power η so that the Hosios condition holds. We follow Shimer (2005) to set the worker's bargaining power η to 0.72 and follow Hagedorn and Manovskii (2008) to set market tightness θ to 0.634. Shimer (2005) calibrates the quarterly job finding rate to 1.35, implying that the matching technology $a = 1.35/0.634^{1-0.72} = 1.534$. Shimer (2012) calculates the job separation rates from the Current Population Survey; he finds that the average job separation rate is 0.0344 during 1948-2007. Hence, we set $\lambda = 0.0344$. The quarterly unemployment rates from 1948 to 2007 are obtained from the Bureau of Labour Statistics, and their average is about $u = 0.056$.

Calibrating unemployment benefits and vacancy posting costs are challenging. Shimer (2005) sets the unemployment benefit to be 0.4. However, Hagedorn and Manovskii (2008) argues that this value is too low as it does not include forgone leisure or home production. They calibrate their model to match the cyclical properties of wages; nevertheless, their result $b = 0.955$ is implausibly large. Mortensen and Nagypal (2007) argue that if $b = 0.955$, the flow surplus of employed workers will be too low. Hence, we follow Hall and Milgrom (2008) and Pissarides (2009) to set $b = 0.71$. This value is larger than 0.4 because it includes the consumption difference between states of employment and unemployment. In Hagedorn and Manovskii (2008), the costs of posting a vacancy include a non-capital hiring cost of 0.110 and an idle capital cost 0.474, amounting to 0.584. c is, therefore, set to be 0.584.

Next in line is the productivity distribution. We follow Zanetti (2011) to assume that a match-specific productivity distribution is log-normally distributed. This calibration strategy coheres with the empirical evidence of Lydall (1968) and Heckman and Sedlacek (1985) in that a wage distribution has a unique interior mode with a log-normal-like skewness. Hence, $\delta \sim \ln N(\mu, \sigma^2)$. We calibrate the mean μ and the standard deviation σ of a productivity distribution together with the equilibrium θ^*, δ^{R*}, and u^*. Given μ and σ, the three variables are pinned down by the three equilibrium conditions (12), (15), and (16). Hence, we solve $\theta^* = 0.634$, $u^* = 0.056$, and the three equilibrium conditions to obtain (μ, σ, u, θ, δ^R). Table 1 summarizes the results.

5.2 Calibrating Entropy Penalty Parameters

This section calibrates the two entropy penalty parameters α and α_v. The calibration procedure closely follows Hansen and Sargent (2008).[9] In brief, α and α_v are first mapped into detection error probabilities for discriminating between an approximating model and a distorted model associated with the corresponding parameter values. Using the detection error probabilities, we pin down α and α_v.

A likelihood ratio test is used to estimate the detection error probability. Consider two alternative models—model A (i.e., an approximating model) and model B (i.e., a distorted model). A worker may observe the samples of realized wages from personal experience, friends, and relatives.[10] The worker uses the approximating model (i.e., model A) that best describes the data generating process to make decisions. We generate $10,000$ samples for each α. In each sample i, 200 observations of wages are generated from the approximating model $\delta \sim \ln N(\mu, \sigma^2)$, where $\mu = 0.4935$ and $\sigma = 0.0822$

[9] Readers who are interested in the details are referred to Chapters 9 and 10 in Hansen and Sargent (2008).
[10] An employer may observe the samples from the resumes of job applicants as well.

Table 1: Endogenous Variables for Calibration

Panel A: Endogenous Variables for Calibration			
Variable	Value	Description	
θ	0.634	Market Tightness	
δ^R	1.662	Reservation Productivity Level	
u	0.056	Unemployment Rate	
Panel B: Calibrated Parameters			
Parameter	Value	Description	Target
μ	0.4935	Mean of a Productivity Distribution	$\theta = 0.634$
σ	0.0822	Standard Deviation	$u = 0.056$
Parameter	Value	Description	Source
r	0.012	Quarterly Interest Rate	Shimer (2005)
λ	0.0344	Separation Rate	Shimer (2012)
b	0.71	Unemployment Benefit	Hall and Milgrom (2008)
η	0.72	Worker's Bargaining Power	Shimer (2005)
a	1.534	Technology of a Matching Function	Shimer (2005)
c	0.584	Cost of Posting a Vacancy	Hagedorn and Manovskii (2008)

are acquired from Table 1. Given a set of sample i, the worker can calculate the likelihood function associated with the approximating model (i.e., L_A^i) for a sample i.

With the approximating model $N(\mu, \sigma^2)$ and the wage equation $w(\delta) = (1-\eta)\delta^R + \eta\delta$, the likelihood function of the approximating model is given by

$$L_A^i = \frac{1}{1 - F(\delta^R)} \left(\prod_{t=1}^{200} \frac{\eta}{(w_t^i - (1-\eta)\delta^R)\sqrt{2\pi}\sigma} e^{-\frac{\left(\ln(w_t^i - (1-\eta)\delta^R) - \ln\eta - \mu\right)^2}{2\sigma^2}} \right),$$

where $F(\cdot)$ follows $\ln N(0.4935, 0.0822^2)$, and w_t^i is the t-th observation in sample i. Other parameters are obtained from Table 1.

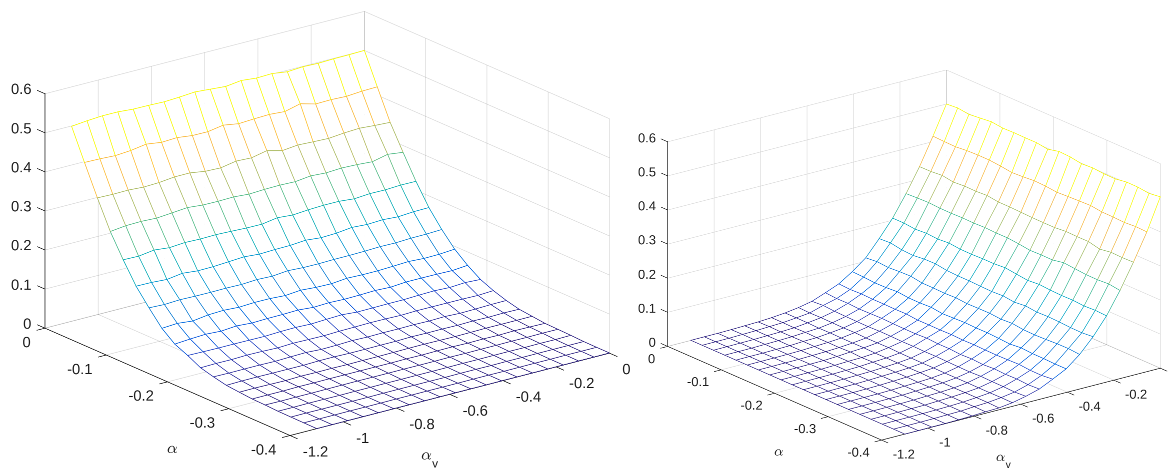
The data generating process is uncertain to the worker. With the fear of model misspecification, the worker makes decisions based on a distorted model B associated with α to maximize the value function under the worst-case scenario. The distorted model is described in equation (2). Given the same set of sample i, the worker associates the distorted model with the likelihood function L_B^i based on the conditional density given by equation (2), in which $\hat{f}(\delta|\delta > \delta^R) = \hat{f}(\delta)/(1 - \hat{F}(\delta^R))$. Noting that $\hat{F}(\cdot)$ is a cumulative distribution function of $\hat{f}(\cdot)$, and $\hat{F}(\delta^R) = e^{\alpha J^U} F(\delta^R) / \int_0^\infty e^{\alpha \max\{J^E(x), J^U\}} dx$.

The test suggests the worker to pick model j iff $L_j^i > L_{-j}^i$. When model j generates the data, the probability of choosing a wrong model $-j$ is given by $\Pr(L_j < L_{-j}|j)$. Hence, the overall detection error probability is defined by

$$(\alpha) = \frac{1}{2}\Big(\Pr(L_A < L_B|A) + \Pr(L_A > L_B|B) \Big).$$

$\Pr(L_j < L_{-j}|j)$ equals $\sum_{i=1}^N I(L_j^i < L_{-j}^i)/N$, where $I(\cdot)$ is an indicator function to count the number of samples in which the worker picks a wrong model and $N = 10,000$ is the number of samples. Since both workers and vacancies could be ambiguity-averse, we compute the detection error probabilities $p(\alpha)$ and $p(\alpha_v)$ using a similar procedure described above.

Figure 3: The Detection Error Probabilities



Note: The left and right panels show the detection error probabilities as a function of workers' and vacancies' entropy penalty parameters.

Figure 3 presents the overall detection error probabilities as a function of two entropy penalty parameters. Given α_v, the overall detection error probability as a function of α is presented in the left panel. Similarly, the right panel presents the overall detection error probability as a function of α_v, given α. Two points deserve mentioning.

First, the overall detection error probabilities $p(\alpha)$ and $p(\alpha_v)$ are decreasing functions. When an entropy penalty parameter is zero, the economic agent has no fear of model misspecification. The agent will select a distorted model that is identical to the approximating one to make decisions. Since the two models cannot be distinguished, the agent will pick the wrong model with a probability of $1/2$. Hence, when $\alpha = 0$ and $\alpha_v = 0$, the corresponding detection error probability is 0.5. As the entropy penalty parameter falls, the agent becomes more concerned with model misspecification and selects a distorted model that is more dissimilar to the approximating one. As such, the two models are more distinguishable and the overall detection error probability declines. Although the two entropy penalty parameters are unbounded, $p(\alpha)$ and $p(\alpha_v)$ decay quickly to zero.

Second, the overall detection probability of workers is weakly related to the ambiguity preference of firms and vice versa. When we fix the value of α in the left panel, the overall detection error probability $p(\alpha)$ is found insensitive to α_v. A similar feature can also be found in the right panel. The results suggest a weak dependence between one entropy penalty parameter and the overall detection error probability associated with another entropy penalty parameter. While the overall detection error probability depends on both the approximating model and the distorted model, the approximating model is independent of the two entropy penalty parameters. In other words, the dependence of workers' distorted model and vacancies' entropy penalty parameter is weak, and vice versa. Although one may think it too strong to assume that the ambiguity preferences of workers and vacancies are common information, the findings of our quantitative exercise suggest that in the equilibrium how unemployed workers formulate their distorted models depends weakly on firms' concern about the robustness of the approximating model, and vice versa.

Here, α can be interpreted as a worker's concern about the robustness of the approximating model with the detection error probability $p(\alpha)$. We follow Croce et al. (2012) to select a five-percent detection error probability. According to Figure 3, workers with $\alpha = -0.18$ are those who pick the distorted model such that the probability of a misspecification error is about five percent. If a reasonable preference for robustness is the rule that functions well for alternative models with the detection error probabilities of five percent or more, $\alpha = -0.18$ will be the choice of the parameter. Similarly, vacancies with $\alpha_v = -0.46$ will pick the distorted model to have the likelihood of a misspecification error equal to five percent.

5.3 Counterfactual Exercises

This section analyzes how ambiguity affects unemployment over business cycles. To quantify the employment effect attributable to ambiguity, we simulate the counterfactual unemployment rate assuming ambiguity neutrality. The difference between the counterfactual and the actual ones quantifies the employment effect of ambiguity.

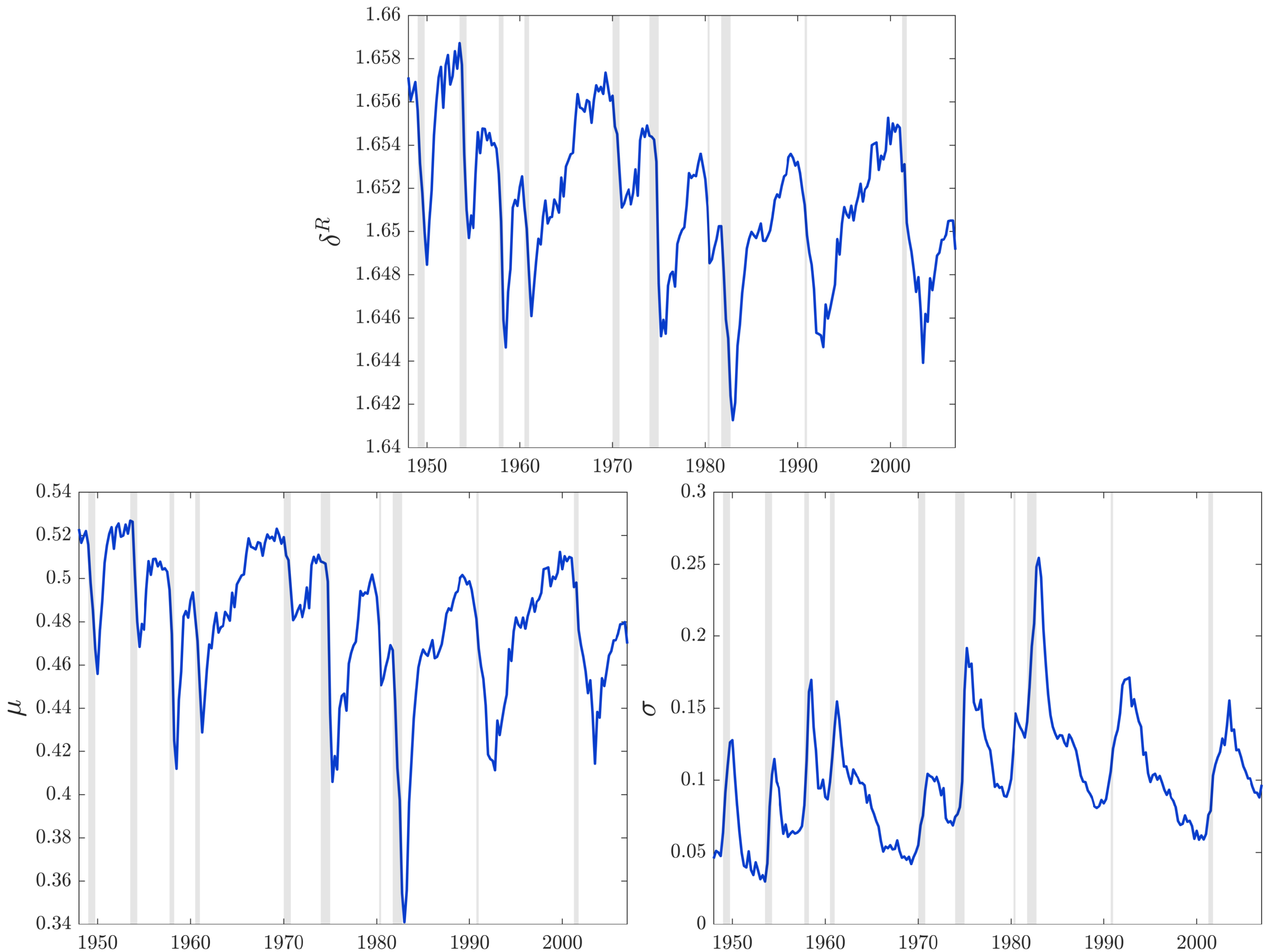
We calibrate the underlying parameters of unemployment to match the US economy in the postwar era. According to equation (12), the steady-state unemployment rate is a function of two job transitional rates—a job-separation rate λ and a job-finding rate $\theta q(\theta)[1 - F(\delta^R)]$. The job-finding rate depends on the productivity distribution function, which is assumed to be log-normally distributed. Hence, the job-finding rate is a function of μ and σ. We will calibrate quarterly u, λ, μ, σ, and δ^R with other parameters being constant as in Table 1.

Here is the calibration procedure. First, we match u to the actual quarterly unemployment rate provided by Shimer (2012). Second, the job-separation rates λ are calibrated to match the quarterly rates provided by Shimer (2012). While Shimer (2012) provides these rates from the first quarter of 1948 to the same quarter of 2007, we restrict our analysis to this period. Third, given the unemployment rate and the job-separation rate, the quarterly job-finding rate can be uniquely pinned down using the steady-state unemployment (12). We calibrate μ, σ, and δ^R to ensure that $\theta q(\theta)[1 - F(\delta^R)]$ equals the quarterly job-finding rate provided by Shimer (2012) and that the two equilibrium conditions (15) and (16) are satisfied. In sum, μ, σ, and δ^R are calibrated such that the unemployment rate and the two job transition rates are identical to the historical rates.

Figure 4 presents the calibrated δ^R, μ, and σ. These calibrated parameters cohere largely with empirical regularities. For example, recessions are believed to be characterized by a negative first-moment shock and a positive second-moment shock. Our calibrated δ^R and μ are high in booms and low in slumps, reflecting the drop in productivity in slumps. Moreover, the calibrated σ is high in slumps, capturing volatility shocks in recession years (Bloom, 2014; Schaal, 2017; Bloom et al., 2018).

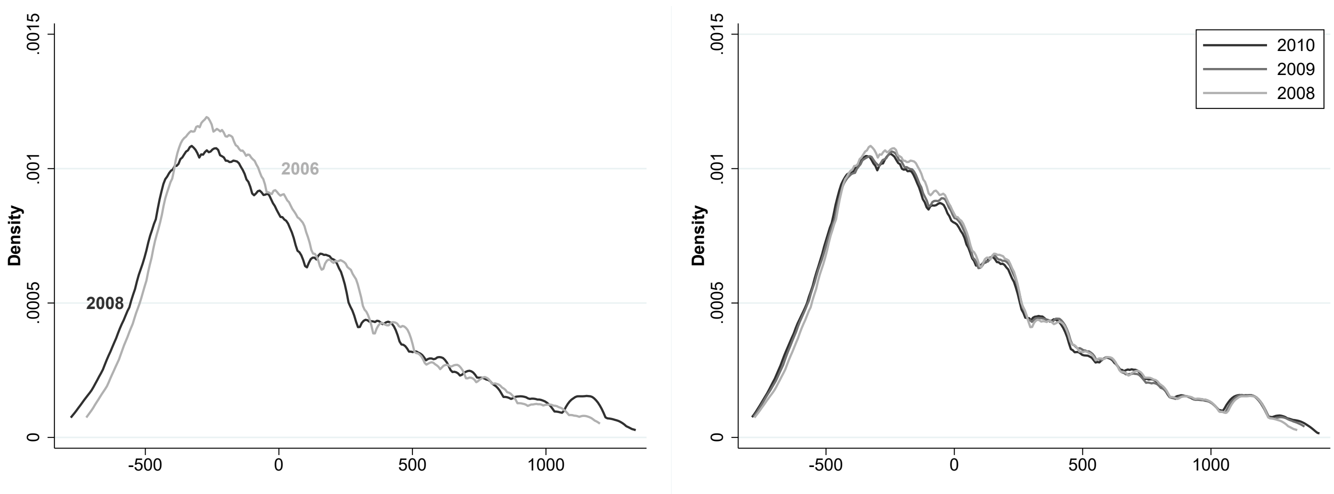
Furthermore, our calibrated productivity distribution is largely consistent with a negative third-

Figure 4: The Calibrated Productivity Distribution Parameters



Note: These figures present the calibrated δ^R, μ, and σ. The gray areas indicate recession years.

Figure 5: The Wage Distribution Function



Note: The demeaned wage distribution in the United States. The distribution is estimated using the Current Population Survey.

moment shock in recession years. That is, a subset of production units performs extraordinarily poorly in slumps, resulting in more less-productive production units. A higher calibrated value of σ implies a higher concentration at the bottom of a log-normal distribution. That is, the bottom of our calibrated productivity distribution is more likely realized in slumps than in booms. This result is largely in line with the recent empirical documentation of skewness shocks: using firm-level data, (Salgado et al., 2019) finds that "*relative to expansion periods, the distribution of employment growth during recessions has a thicker left tail, whereas the right tail exhibits little change, indicating an increase in dispersion that is mostly due to a widening left tail.*"

To further provide evidence on the skewness shocks, we use individual-level data to plot the wage distribution before and after recession years. Using the US Current Population Survey, Figure 5 presents the demeaned wage distribution. Clearly, relative to 2006, the wage distribution has a thicker left tail in 2008, the beginning of recession years, leaving the right tail nearly unchanged. A similar change cannot be found after 2008. In other words, the left tail of the wage distribution tends to be thicker during the entire recession years, providing another support on the skewness shocks in downturns.

We proceed to quantify the employment effect of ambiguity. Thus far, agents have been assumed to be ambiguity-averse in the calibrated economy. Now, we simulate the economy in which agents became ambiguity-neutral by setting the two entropy penalty parameters (i.e., α and α_v) to zero. As such, we can construct counterfactual quarterly unemployment rates in the absence of ambiguity aversion. The difference between the historical unemployment rate and the counterfactual unemployment rate will capture the employment effect of ambiguity. In this exercise, we allow the job-separation rate λ and the mean μ and standard deviation σ of the productivity distribution to be time-varying. Given the quarterly λ, μ, and σ and other time-invariant exogenous variables from Table 1, we obtain δ^{R*} and θ^* by solving equations (15) and (16) simultaneously. Together with μ and σ, the resulting δ^{R*} and θ^* will return us with quarterly job-finding rates according to $\theta q(\theta)[1-F(\delta^R)]$. The historical job-separation rate and the simulated job-finding rate jointly determine the unemployment rate in each quarter based on equation (12). Figure 6 demonstrates the actual and counterfactual unemployment rates.

Several important points emerge immediately. First, ambiguity preferences depress unemployment. Figure 6 shows that the counterfactual unemployment rate (the dashed line) is consistently higher than the actual one (solid line). If both workers and vacancies became ambiguity-neutral, the unemployment rate would have increased throughout the entire period of examination. According to Proposition 2, unemployment decreases with the degree of workers' ambiguity aversion. However, it is analytically unclear whether unemployment increases or not in response to stronger vacancies' ambiguity preference. Hence, it is uncertain how the removal of the ambiguity from both workers and vacancies affects unemployment. This quantitative analysis suggests that but for ambiguity preferences, the historical unemployment rate would have been higher in the postwar era.

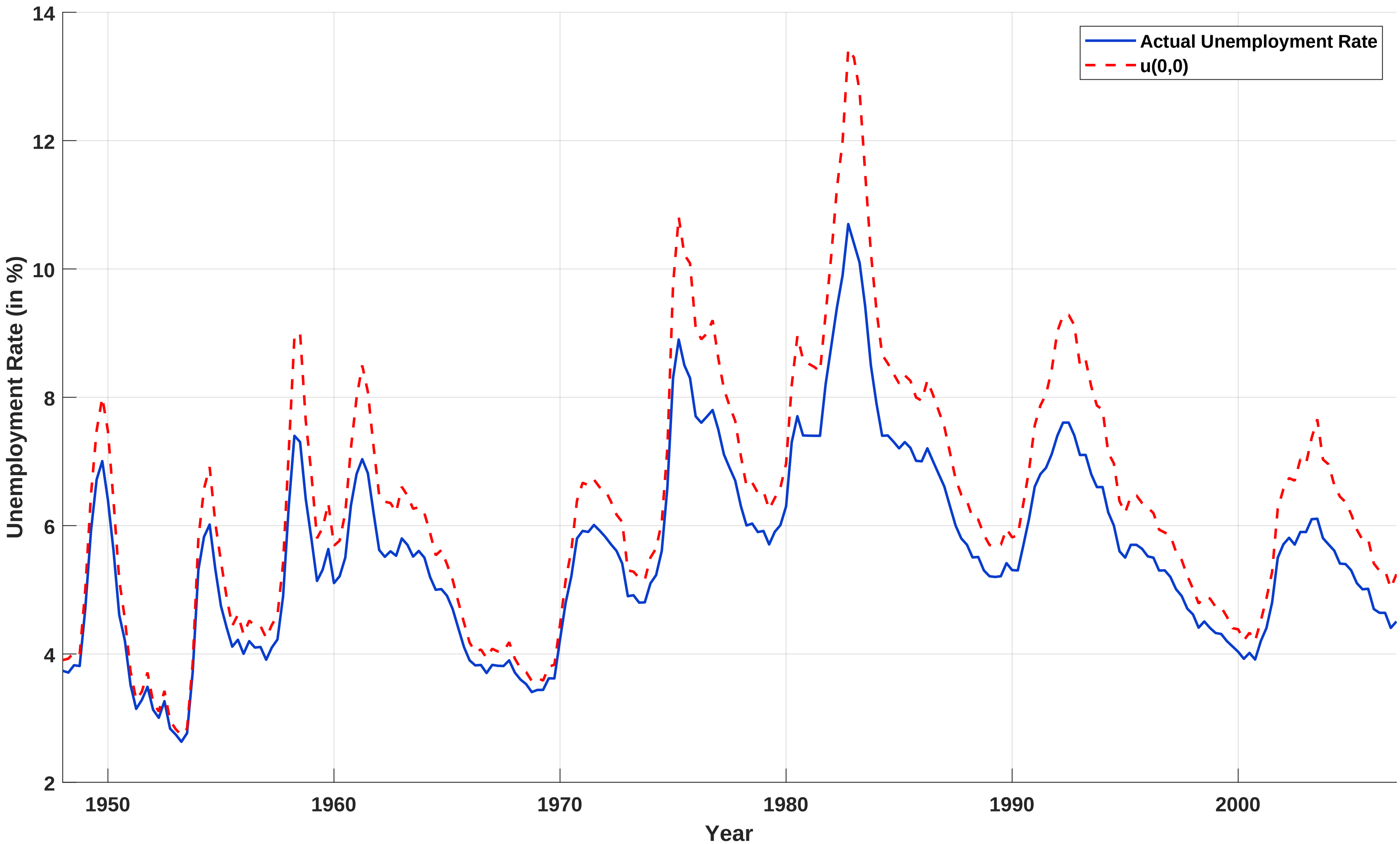
Second, the employment effect of ambiguity is countercyclical. The ambiguity preference could distort our beliefs on a productivity distribution, motivating unemployed workers to accept job offers. In booms, such the distortion is minimal; hence, the actual and counterfactual unemployment rates are nearly identical. In slumps, unemployed workers are willing to accept job offers with poor match qualities regardless of ambiguity preference. The distortion further increases their willingness to accept low-quality job offers. As a result, the ambiguity shortens job search processes and depresses unemployment, especially in recession years.

Third, the employment effect of ambiguity could be significant. While the divergence between the actual and counterfactual unemployment rates is nearly zero in booms, it could reach as high as 19 percent of the actual ones in recession years. As such, averaging the employment effect across times may mask the important role ambiguity plays in the labour market. By uncovering the employment effects over business cycles, our quantitative analysis reveals the empirical significance of ambiguity, calling for attention to ambiguity in labour market policy formulation.

What makes the employment effect of ambiguity countercyclical? This question is informative: not only does it give a lesson on the role ambiguity plays in the labour market, but it also enhances our understanding of the functioning of labour markets over business cycles. We investigate whether the cyclical property of the employment effect results mainly from the two parameters of the productivity distribution function (i.e., μ and σ).

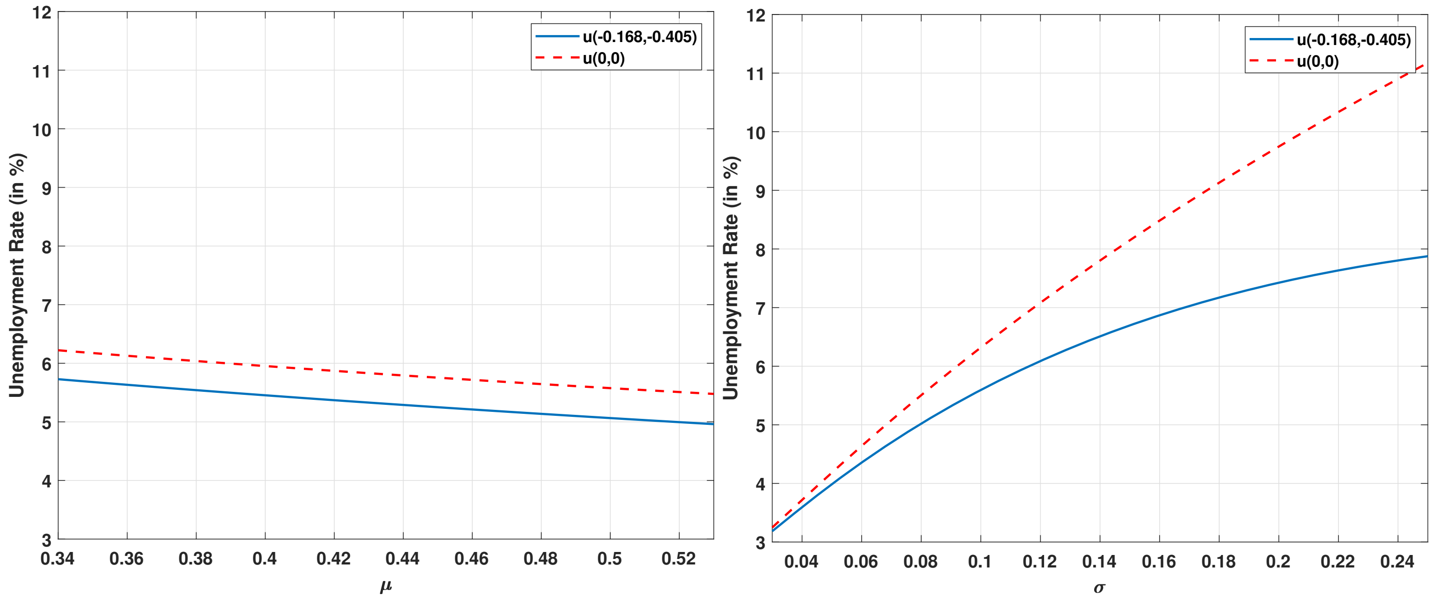
We demonstrate the relationship between the unemployment rate and the two parameters of the calibrated productivity distribution function in Figure 7. To begin with, workers and vacancies are assumed to be ambiguity-averse (i.e., $\alpha = -0.168$ and $\alpha_v = -0.405$). We allow μ to vary from its minimum to maximum in the calibrated US economy in the postwar era (i.e., between 1948 and 2007). That is μ ranges between its smallest and highest values in Figure 6. For each μ, we solve θ^* and δ^{R*} such that the two variables satisfy the two equilibrium conditions (15) and (16), with other parameters being fixed at their averages in the postwar era. With the two endogenous variables, u is solved using the steady-state unemployment (12). We present the relationship between u and μ under ambiguity aversion (i.e., the solid line) in Figure 7. We repeat the analysis assuming agents to be ambiguity-neutral (i.e., $\alpha = 0$ and $\alpha_v = 0$) and present the relationship between u and μ under ambiguity neutrality (i.e., the dashed line)

Figure 6: The Actual and Counterfactual Unemployment Rates in the United States



Notes: The solid line represents the actual quarterly unemployment rate in US. The dash line shows the counterfactual unemployment rate when both workers and vacancies became ambiguity-neutral.

Figure 7: The Effects of μ and σ on Unemployment



Notes: The left (right) panel demonstrates the relationship between μ (σ) and unemployment. The solid and dash lines demonstrate the corresponding relationship in the US economy with ambiguity aversion and without ambiguity aversion, respectively. Other variables are computed using their averages in the calibration exercise in Figure 6. Along the horizontal axes, μ and σ vary from its minimum to maximum in the calibrates US economy presented in Figure 6.

in the same figure. Likewise, we replicate the analysis and present a comparable relationship between u and σ in Figure 7.

The analysis enhances our understanding of the productivity shocks over business cycles. The results suggest that the employment effect of ambiguity is countercyclical not because of the first-moment shocks. According to Figure 7, the unemployment rates fall with μ. While the average productivity level is mainly governed by μ, the result is consistent with the traditional thought on the procyclicality of unemployment. It is noteworthy that the two lines are nearly parallel, suggesting that the relationship between the unemployment rate and the average productivity level is insensitive to ambiguity. Hence, ambiguity aversion does not catalyze the unemployment effect of negative first-moment shocks, and (ii) the first-moment shock does not explain the countercyclicality of the employment effect of ambiguity as shown in Figure 6.

Our analysis suggests that the countercyclicality of the employment effect is largely related to the skewness shocks. Generally, an increase in σ causes the standard deviation and skewness of a log-normal distribution, including the productivity distribution function, to rise. In a recession, volatility shocks spread productivity out and skewness shocks make the productivity distribution function more left-leaning. As such, the skewness shocks increase the likelihoods assigning to lower productivity levels to be realized in an approximation model. Meanwhile, the ambiguity preferences further assign higher likelihoods to lower productivity levels in a distorted model. The two effects reinforce each other. Since σ is larger in recession years (See Figure 4), the skewness shock intensifies the employment effect of ambiguity in recession (See Figure 7).

Our findings share the same notion with Epstein and Schneider (2008) in that ambiguity-averse agents react more strongly to negative shocks than positive shocks. Agents tend to pick the distorted model that departs more from the approximating model in recession than in expansion. But for the ambiguity, the unemployment rate would be larger in recession. Our quantitative analysis contributes to the literature by linking ambiguity about productivity to the close relationship between unemployment and skewness shocks. This linkage, though important in understanding unemployment fluctuations, has not been addressed in previous formal models.

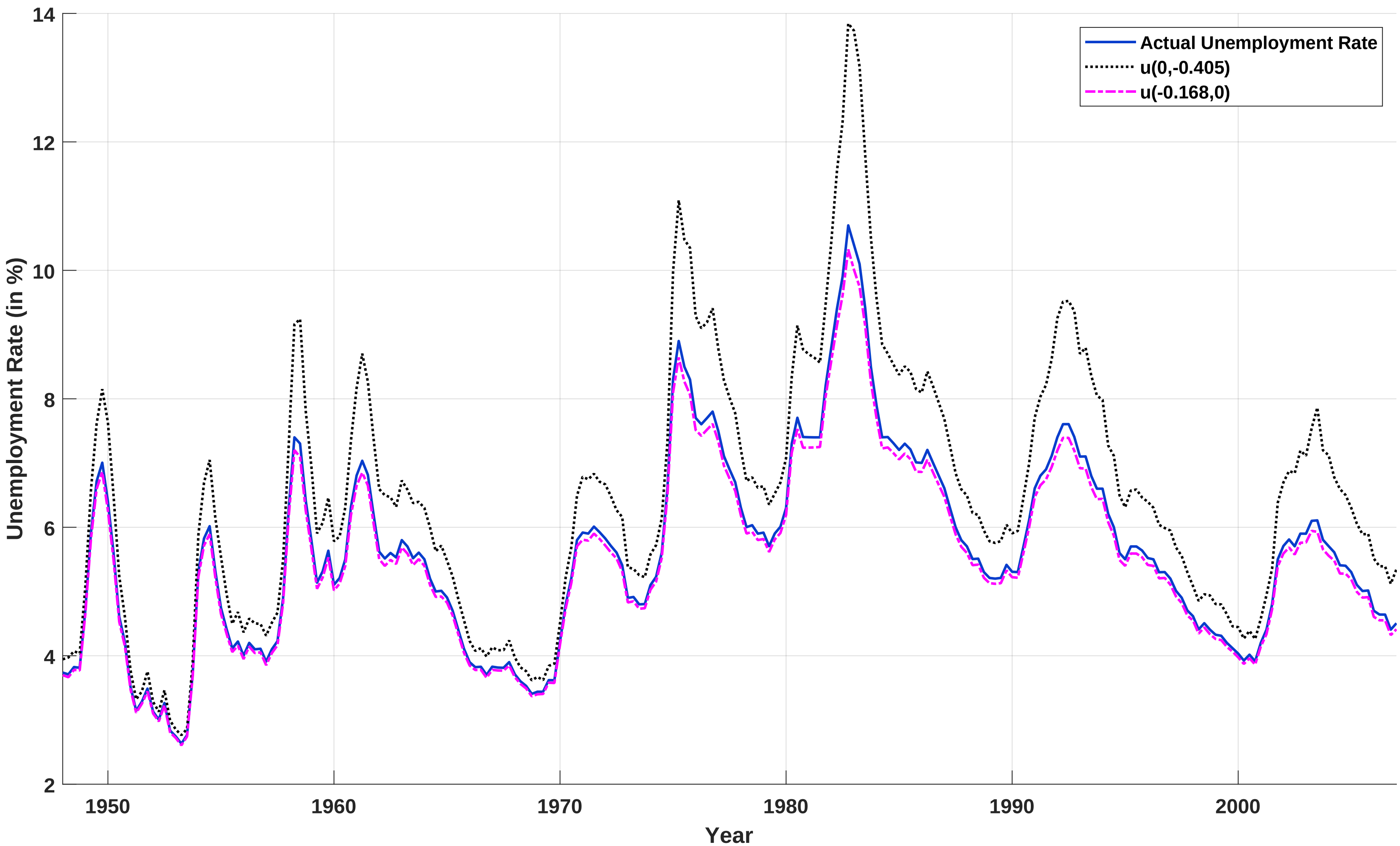
Another natural question emerges: is the significant employment effect driven mainly by workers' or vacancies' ambiguity preferences? If the propagation of labour market information could help the public reduce ambiguity, should we allocate resources to pass labour market information to employers or employees? To answer this question, we replicate the above analysis by forcing workers to be ambiguity-neutral and allowing vacancies to be ambiguity-averse. The counterfactual unemployment rate $u(0, -0.405)$ is presented in Figure 8. Analogously, we simulate the economy in which workers are ambiguity-averse and vacancies are ambiguity-neutral and present the counterfactual unemployment rate $u(-0.168, 0)$ in Figure 8.

The results suggest that the employment effects of workers' and vacancies' ambiguity preferences are qualitatively and quantitatively different. When workers became ambiguity-neutral, the unemployment rate would be substantially higher than the historical one. The effect is substantial for two reasons. First, the workers' ambiguity preference distorts their beliefs about a productivity distribution function, motivating them to make decisions based on a distorted model with a lower productivity level. With such a distorted model, unemployed workers are willing to accept low-quality job offers. This primary effect depresses unemployment at the expense of match qualities.

Meanwhile, the distorted belief lowers their outside option value. As a result, a lower bargained wage is sufficient to compensate workers. From the vacancies' perspective, a lower wage means lower labour costs and higher flow profits. The higher flow profits incent the supply of vacancies. This general equilibrium effect further depresses unemployment. In sum, the two effects jointly contribute to the substantial decrease in the unemployment rate.

When vacancies became ambiguity-neutral, the unemployment rate would be slightly smaller than the historical one. Similar to workers, vacancies make decisions according to a distorted model with a lower productivity level. As a result, vacancies are willing to sign contracts with low-quality matches. This effect depresses unemployment. With a lower productivity level, the distorted model returns vacancies with lower flow profits. As such, the supply of vacancies decreases; it takes unemployed workers a long time to find jobs, lengthening the spell of unemployment. While it is analytically unclear about the relative size of the two competing forces, our quantitative analysis indicates that the former effect is dominated, explaining why the historical unemployment rate is higher than the unemployment rate of an economy in which vacancies were ambiguity-neutral. Additionally, the quantitative analysis suggests that the sizes of the two competing forces are so close that they nearly cancel each other as to why the difference between the counterfactual unemployment rate $u(-0.168, 0)$ and the historical one is tiny over business cycles—the employment effect of vacancies' ambiguity preference is weak.

Figure 8: The Employment Effects of Workers' and Vacancies' Ambiguity



Note: The dot (dash-dot) line shows the counterfactual unemployment rate when workers (vacancies) became ambiguity-neutral.

An interesting policy dilemma concerning labour market information emerges. According to the quantitative analysis, the revelation of labour market information to workers increases unemployment, and the effect is substantial. Therefore, passing labour market information to workers hurts an economy in slumps by worsening unemployment. In principle, the same tool cools an overheated economy in booms; nevertheless, the employment effect of ambiguity is small in booms.

In contrast, the removal of vacancies' ambiguity depresses unemployment, but the effect is tiny. Quite often, policymakers may want to encourage the creation of job vacancies by providing vacancies with abundant labour market information. The findings of the quantitative analysis suggest that it helps, but policymakers are simply *busy doing nothing*: the policy is ineffective.

6 Conclusion

This paper studies how ambiguity towards a productivity distribution affects the labour market. It makes three primary contributions.

First, it constructs an analytically tractable search-theoretical model featuring ambiguity preferences. It generalizes the DMP model by incorporating the Hansen and Sargent (2008) type of ambiguity preferences. Our analytically tractable model allows us to uncover major mechanisms through which ambiguity affects labour market outcomes. While Nishimura and Ozaki (2004) incorporates Knightian uncertainty into a job-seeker problem, this paper studies both workers' and vacancies' problems, completing the search process between workers and firms.

Second, we quantify the unemployment attributable to the ambiguity. We find that the ambiguity depresses the American unemployment rate, and the effect is much larger in slumps than in booms. We also discover that the countercyclicality of the employment effect of ambiguity is largely related to skewness shocks. Our analysis complements a series of influential papers that study the relationship between unemployment and the productivity distribution (Bloom et al., 2007; Bloom, 2009; Schaal, 2017; Bloom et al., 2018; Salgado et al., 2019). This paper contributes to this literature by showing the linkage between ambiguity and the skewness of the productivity distribution, enhancing our understanding of the labour market over business cycles.

Third, we discuss possible policy implications concerning labour market information. We find that the revelation of labour market information to workers will increase unemployment because of ambiguity. On the other hand, passing the information to employers will decrease unemployment, but the effect is tiny.

7 Appendix: The Proofs of Propositions

7.1 Proof of Equation (2)

To solve the minimization problem (1), we can write a Lagrangian function as follows.

$$L = b+\beta\mathbb{E}_x\left[\theta q(\theta)\left(m(x)\max\{J^E(x),J^U\}-\frac{1}{\alpha}m(x)\ln m(x)\right)+(1-\theta q(\theta))J^U\right]+\lambda\left[1-\int m(x)dF(x)\right].$$

The first order condition gives us

$$\theta q(\theta)\left[\max\{J^E(x),J^U\}-\frac{1}{\alpha}-\frac{\ln m(x)}{\alpha}\right]-\lambda=0.$$

Rearranging terms gives us

$$m(x)=e^{\alpha\max\{J^E(x),J^U\}}e^{-1-\frac{\alpha\lambda}{\theta q(\theta)}}.$$

Integrating both sides over $\mathbb{R}$ and rearranging terms give us

$$\begin{aligned}\int m(x)dF(x) &= \int e^{\alpha\max\{J^E(x),J^U\}}dF(x)e^{-1-\frac{\alpha\lambda}{\theta q(\theta)}}\\ e^{1+\frac{\alpha\lambda}{\theta q(\theta)}} &= \int e^{\alpha\max\{J^E(x),J^U\}}dF(x).\end{aligned}$$

Hence, we have

$$m(\delta)=\frac{e^{\alpha\max\{J^E(\delta),J^U\}}}{\int e^{\alpha\max\{J^E(x),J^U\}}dF(x)}.$$

7.2 Derivations of Equations (15) and (16)

Substituting the wage equation (9) in equation (3) yields

$$J^E(\delta)=(1-\eta)\beta rJ^U+\eta\delta+\beta\left[\lambda J^U+(1-\lambda)J^E(\delta)\right].$$

Rearranging terms gives

$$J^E(\delta)-J^U=\frac{\eta(1+r)}{(r+\lambda)}(\delta-\delta^R)$$

Substituting this $J^E(\delta)-J^U$ into equation (13) yields equation (15). Equation (16) can be derived in a similar way.

7.3 Proof of Proposition 2

Assume that $\delta^R\geq b$ is finite. So, $0<F(\delta^R)\leq 1$. Define

$$B(\alpha)\equiv F(\delta^R)+\int_{\delta^R}^{\infty}e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}dF(x)$$

Its first- and second-order partial derivative are given by

$$B'(\alpha) = \frac{\eta(1+r)}{r+\lambda}\int_{\delta^R}^{\infty}(x-\delta^R)e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}dF(x)$$

and

$$B''(\alpha) = \left[\frac{\eta(1+r)}{r+\lambda}\right]^2\int_{\delta^R}^{\infty}(x-\delta^R)^2e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}dF(x).$$

Next, we define

$$\tilde{B}(\alpha) \equiv \frac{1}{\alpha}\ln B(\alpha) > 0 \text{ for all finite } \alpha \leq 0,$$

where $\tilde{B}(\alpha)$ is the last term on the R.H.S. of equation (15). We will show that $\tilde{B}'(\alpha) > 0$ for all $\alpha \leq 0$. The first- and the second-order partial derivatives of $\tilde{B}(\alpha)$ are given by

$$\tilde{B}'(\alpha) = \frac{1}{\alpha}\left(-\frac{1}{\alpha}\ln B(\alpha) + \frac{B'(\alpha)}{B(\alpha)}\right)$$

and

$$\tilde{B}''(\alpha) = -\frac{2}{\alpha}\tilde{B}'(\alpha) + \frac{1}{\alpha}\left(\frac{B''(\alpha)}{B(\alpha)} - \frac{B'(\alpha)^2}{B(\alpha)^2}\right).$$

Lemma 1.

$$\frac{B''(\alpha)}{B(\alpha)} - \frac{B_1(\alpha)^2}{B(\alpha)^2} > 0$$

Proof. Define a function $h(x,\alpha)$ as

$$h(x,\alpha) = \begin{cases} 1 & \text{if } 0 \leq x < \delta^R \\ e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)} & \text{if } x \geq \delta^R \end{cases}$$

Hence, $B(\alpha) = \int_0^\infty h(x,\alpha)f(x)dx$. For any $\alpha \leq 0$, we can define another function

$$g(x,\alpha) \equiv \frac{h(x,\alpha)}{B(\alpha)}f(x).$$

$g(x,\alpha)$ is positive because $h(x,\alpha) > 0$ and $B(\alpha) > 0$ for all $\alpha \leq 0$. Hence, $\int_0^\infty g(x,\alpha) = 1$ and $g(x,\alpha)$ is a probability density function.

$$\frac{B'(\alpha)}{B(\alpha)} = \frac{1}{B(\alpha)}\int_0^\infty \frac{\partial h(x,\alpha)}{\partial\alpha}f(x)dx = \frac{\eta(1+r)}{r+\lambda}\int_0^\infty \max\{x-\delta^R,0\}g(x,\alpha)dx$$

$$\frac{B''(\alpha)}{B(\alpha)} = \frac{1}{B(\alpha)}\int_0^\infty \frac{\partial^2 h(x,\alpha)}{\partial\alpha^2}f(x)dx = \left[\frac{\eta(1+r)}{r+\lambda}\right]^2\int_0^\infty \Big(\max\{x-\delta^R,0\}\Big)^2 g(x,\alpha)dx$$

Therefore,

$$\frac{B''(\alpha)}{B(\alpha)} - \left(\frac{B'(\alpha)}{B(\alpha)}\right)^2 = \left[\frac{\eta(1+r)}{r+\lambda}\right]^2 Var_x\Big(\max\{x-\delta^R,0\}\Big) > 0 \text{ for all finite } \delta^R,$$

where the variance is evaluated with respect to the density function $g(x, \alpha)$. $\square$

Now, we are ready to show that $\tilde{B}'(\alpha) > 0$ for all $\alpha \leq 0$. First, notice that $\lim_{\alpha \to 0} \tilde{B}(\alpha) > 0$ and $\lim_{\alpha \to -\infty} \tilde{B}(\alpha) = 0$.

Second, there exists no $\alpha_0 \leq 0$ such that $\tilde{B}'(\alpha_0) = 0$ and $\tilde{B}''(\alpha_0) \geq 0$. Otherwise, it contradicts to Lemma 1. Hence, if there exists $\alpha_0 \leq 0$ such that $\tilde{B}'(\alpha_0) = 0$, $\tilde{B}''(\alpha_0) < 0$.

Third, there exists no $\alpha_0 \leq 0$ such that $\tilde{B}'(\alpha_0) = 0$ and $\tilde{B}''(\alpha_0) < 0$. Suppose not. Then $\lim_{\alpha \to 0^-} \tilde{B}(\alpha) < 0$ because $\lim_{\alpha \to 0} \tilde{B}(\alpha) > 0$ and $\lim_{\alpha \to -\infty} \tilde{B}(\alpha) = 0$.

By L'Hopital's rule, $\lim_{\alpha \to 0^-} \tilde{B}(\alpha) = \lim_{\alpha \to 0^-} B'(\alpha)/B(\alpha)$. Hence, we can apply the L'Hopital's rule to evaluate $\lim_{\alpha \to 0^-} \tilde{B}'(\alpha)$. Hence, we have

$$\lim_{\alpha \to 0^-} \tilde{B}'(\alpha) = \lim_{\alpha \to 0^-} \left[-\tilde{B}'(\alpha) + \frac{B''(\alpha)}{B(\alpha)} - \left(\frac{B'(\alpha)}{B(\alpha)} \right)^2 \right],$$

and thus

$$\lim_{\alpha \to 0^-} \tilde{B}'(\alpha) = \lim_{\alpha \to 0^-} \frac{1}{2} \left[\frac{B''(\alpha)}{B(\alpha)} - \left(\frac{B'(\alpha)}{B(\alpha)} \right)^2 \right] > 0.$$

A contradiction arises. Therefore, there exists no $\alpha \leq 0$ such that $\tilde{B}'(\alpha) = 0$. Since $\lim_{\alpha \to 0} \tilde{B}(\alpha) > 0$ and $\lim_{\alpha \to -\infty} \tilde{B}(\alpha) = 0$, $\tilde{B}'(\alpha) > 0$ for all $\alpha \leq 0$.

Therefore, the partial derivative of the R.H.S. of equation (15) with respect to α is positive. A similar argument can be used to show that the partial derivative of the R.H.S. of equation (16) with respect to α_v is positive. Applying Cramer's rule to equations (15) and (16),

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix} \begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} -d\alpha_v \\ -d\alpha \end{pmatrix}$$

Define $A \equiv \det \begin{pmatrix} - & - \\ + & - \end{pmatrix}$.

$$\frac{d\theta}{d\alpha_v} = \frac{\det \begin{pmatrix} - & - \\ 0 & - \end{pmatrix}}{A} > 0, \quad \frac{d\delta^R}{d\alpha_v} = \frac{\det \begin{pmatrix} - & - \\ + & 0 \end{pmatrix}}{A} > 0,$$

$$\frac{d\theta}{d\alpha} = \frac{\det \begin{pmatrix} 0 & - \\ - & - \end{pmatrix}}{A} < 0, \text{ and } \frac{d\delta^R}{d\alpha} = \frac{\det \begin{pmatrix} - & 0 \\ + & - \end{pmatrix}}{A} > 0$$

Using the wage equation (9), simple algebra gives

$$\frac{dw(\delta)}{d\alpha} = (1 - \eta) \frac{d\delta^R}{d\alpha} > 0, \text{ and } \frac{dw(\delta)}{d\alpha_v} = (1 - \eta) \frac{d\delta^R}{d\alpha_v} > 0.$$

7.4 Proof of Proposition 4

$G \succeq_{FOSD} F$ iff $\int h(x)dG(x) \leq \int h(x)dF(x)$ for any non-increasing function $h(x)$. For any $\delta^R \in \mathcal{R}_+$, we define two non-increasing functions:

$$h(x) = \begin{cases} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}, & \text{if } \delta > \delta^R; \\ 1, & \text{otherwise.} \end{cases}, \ \tilde{h}(x) = \begin{cases} e^{\frac{\alpha_v(1-\eta)(1+r)}{r+\lambda}(x-\delta^R)}, & \text{if } \delta > \delta^R; \\ 1, & \text{otherwise.} \end{cases}$$

Hence, equations (15) and (16) can be written as follows:

$$\begin{aligned} \delta^R &= b + \frac{\beta}{\alpha}\ln\left(\int_0^\infty h(x)dF(x)\right) \\ c &= \frac{\beta}{\alpha_v}\ln\left(\int_0^\infty \tilde{h}(x)dF(x)\right) \end{aligned}$$

Define a surjective function $A(\delta^R; a) : \mathcal{R}_+ \times \mathcal{R} \to \mathcal{R}$, with $\partial A(\delta^R; a)/\partial a > 0$. Notice that for any distribution G, there exist a_G and $\tilde{a}_G$ that satisfy the following equations.

$$\begin{aligned} \int_0^\infty h(x)dG(x) + A(\delta_F^R; a_G) &= \int_0^\infty h(x)dF(x) \\ \int_0^\infty \tilde{h}(x)dG(x) + A(\delta_F^R; \tilde{a}_G) &= \int_0^\infty \tilde{h}(x)dF(x) \end{aligned}$$

where δ_F^R is the reservation productivity under the distribution F. Notice that a_G and $\tilde{a}_G$ are unique due the monotonicity of $A(\delta^R; a)$ with respect to a. Without loss of generality, we set $a_F = \tilde{a}_F = 0$ so that $A(\delta^R; 0) = 0$. For any $G \succeq_{FOSD} F$ where $G \neq F$, either $a_G > 0$ or $\tilde{a}_G > 0$ (or both). Consider a distribution $G_0 \succeq_{FOSD} F$ with $a_{G_0} = \tilde{a}_{G_0}$ slightly above zero. Thus, the preceding two equations can be written as follows:

$$\begin{aligned} \delta^R &= b + \frac{\beta}{\alpha}\ln\left(\int_0^\infty h(x)dF(x) - A(\delta^R; a_{G_0})\right) \\ c &= \frac{\beta}{\alpha_v}\ln\left(\int_0^\infty \tilde{h}(x)dF(x) - A(\delta^R; \tilde{a}_{G_0})\right) \end{aligned}$$

To investigate the impact of changing the distribution from F to G_0, it is equivalent to investigate the impact of changing a from 0 to $a_{G_0} = \tilde{a}_{G_0}$. Hence, we apply Crammer's rule to the above two equations,

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix}\begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} -da \\ -da \end{pmatrix}$$

$$\frac{d\delta^R}{da} = \frac{\det\begin{pmatrix} - & - \\ + & - \end{pmatrix}}{A} > 0, \ \frac{d\theta}{da} = \frac{\det\begin{pmatrix} - & - \\ - & - \end{pmatrix}}{A}$$

Thus, $\delta_G^R > \delta_F^R$ if $G \succeq_{FOSD} F$. Hence, $w_G(\delta) = (1-\eta)\delta_G^R + \eta\delta > (1-\eta)\delta_F^R + \eta\delta = w_F(\delta)$ for all $\delta \geq \delta_G^R$. Similarly, applying Crammer's rule to the cases ($a_G > 0$, $\tilde{a}_G = 0$) and ($a_G = 0$, $\tilde{a}_G > 0$) gives the same result.

7.5 Proof of Proposition 5

Applying Cramer's rule to equations (15) and (16),

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix} \begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} +dc \\ -db \end{pmatrix} \tag{17}$$

Denote $A = det \begin{pmatrix} - & - \\ + & - \end{pmatrix} > 0$.

$$\frac{d\theta}{dc} = \frac{\det \begin{pmatrix} + & - \\ 0 & - \end{pmatrix}}{A} < 0, \ \frac{d\delta^R}{dc} = \frac{\det \begin{pmatrix} - & + \\ + & 0 \end{pmatrix}}{A} < 0 \tag{18}$$

$$\frac{d\theta}{db} = \frac{\det \begin{pmatrix} 0 & - \\ - & - \end{pmatrix}}{A} < 0, \ \frac{d\delta^R}{db} = \frac{\det \begin{pmatrix} - & 0 \\ + & - \end{pmatrix}}{A} > 0 \tag{19}$$

7.6 Proof of Proposition 6

If $q_1 = \varphi q$ where $\varphi \in (0,1)$, then $p_1 = \varphi p$. With new matching technology, equations (15) and (16) are written as

$$\delta^R = b + \frac{\beta}{\alpha} \ln \left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha\varphi\theta q(\theta)\eta(1+r)}{r+\lambda}(x-\delta^R)} dF(x) \right) \tag{20}$$

$$c = \frac{\beta}{\alpha_v} \ln \left(F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha_v\varphi q(\theta)(1-\eta)(1+r)}{r+\lambda}(x-\delta^R)} dF(x) \right) \tag{21}$$

Applying Crammer's rule to the above two equations,

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix} \begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} -d\varphi \\ -d\varphi \end{pmatrix} \tag{22}$$

$$\frac{d\delta^R}{d\varphi} = \frac{\det \begin{pmatrix} - & - \\ + & - \end{pmatrix}}{A} > 0, \ \frac{d\theta}{d\varphi} = \frac{\det \begin{pmatrix} - & - \\ - & - \end{pmatrix}}{A} \tag{23}$$

Thus, $dw(\delta)/d\varphi > 0$ for all $\delta \geq \delta^R$.

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