

ON THE DISTRIBUTIONS OF CONSUMPTION, INCOME, AND WEALTH: THE INTERPLAY BETWEEN EXTERNALITY TAXES AND INCOME TAXES

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Abstract

This paper answers (i) how households and firms behave as the households grow more concerned with negative production externalities such as pollution, (ii) how externality and income taxes should respond to this increasing concern, and (iii) how the two taxes affect the distributions of the consumption, income, and wealth. We develop a general equilibrium model featuring production externality in utility and household heterogeneity in productivity. We analytically show that the increasing concern could induce more externalities, and an externality tax could increase consumption and income inequalities. Our numerical analysis, extending the model to a DSGE framework, finds that (i) optimal taxation and the associated distributional consequences largely depend on policy constraints, and (ii) the impacts of these constraints become more significant if externalities arouse more public concern.

Keywords: Production Externalities; Optimal Taxation; Policy Constraints; Consumption, Income, and Wealth Distributions.

JEL Classification Numbers: D3, E2, H2, Q5.

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1 Introduction

The public becomes increasingly conscious of negative production externalities such as pollution. These concerns have stimulated interest in a number of potential policy responses including externality taxes. “*The distributional impacts of [these] policies clearly are highly relevant to social welfare, and such impacts often critically influence political feasibility*” (Bovenberg et al., 2005). Yet we know little about the distributional consequences of these policies. This paper purposes to answer (i) how households and firms behave as the households grow more concerned with negative production externalities, (ii) how externality and income taxes should respond to this increasing concern, and (iii) how the two taxes affect the distributions of consumption, income, and wealth.

This paper contributes to the analytical and empirical literatures on externality taxes. It develops an analytical general equilibrium model featuring production externality in utility and household heterogeneity in productivity, both of which are directly relevant to the problem at hand. While an extensive literature assumes a utility function separable in externality, this paper relaxes this assumption: each household has a nonseparable utility function in goods, leisure, and externality. Such an extension allows us to uncover mechanisms through which increasing concerns over production externalities and the two taxes affect aggregate variables such as consumption, income, externality, and two critically important yet often ignored variables—the distributions of consumption and income—in a stylized analytical model.¹

Moreover, this paper quantitatively characterizes the optimal externality and income taxes and investigates their distributional consequences via the Aiyagari (1994) model, the main workhorse in the heterogeneous agent literature. Our paper thus combines the strengths of analytical and numerical approaches: while a stylized analytical model uncovers major mechanisms at play, a numerical model measures the empirical significance of these mechanisms in a more realistic model framework.

Our analytical results indicate that externality could increase as the public grows more concerned with externality. An increasing concern over production externalities reduces the utility derived from consumption, raising the marginal utility of consumption and thus both

¹We interpret the increase in households’ disutility towards negative production externalities as an increasing concern over negative production externalities.

household consumption and labor supply.² Aggregate consumption increases in a general equilibrium; firms respond by producing more goods and thus externalities. Since more-productive households tend to work more, their marginal utilities of leisure are larger: the labor supply of a more-productive household is less sensitive. In response to the increasing concern, the labor supply of a more-productive household increases less; therefore, the Gini coefficients of labor income and thus consumption decline.

Our analytical model predicts that an externality tax reduces externality because of the increase in the marginal cost of externality. Such a reduction in externality increases the utility derived from consumption, decreasing the marginal utility of consumption and thus both household consumption and labor supply. In a general equilibrium, aggregate consumption, aggregate output, and externality drop. As the labor supply of a more-productive household is less responsive, this externality tax reduces the labor supply of a more-productive household less. Consequently, the externality tax increases consumption and income inequalities.

Another contribution of this paper is its numerical investigation of optimal externality and income tax policies under realistic policy constraints. Our numerical results show that under no policy constraint, the optimal externality tax rises with the increasing concern, and the optimal income tax hardly reacts to this concern. [Bovenberg and Goulder \(1996\)](#) show that policy constraints could substantially affect the optimal externality tax rate.³ This paper contributes to the literature by showing that neglecting these policy constraints could lead to the prescriptions of the externality and income tax codes that deviate substantially from the optimum and to the distributional consequences on consumption, income, and wealth that could be qualitatively and quantitatively different. Furthermore, this paper finds that the impacts of these constraints become more significant as households grow more concerned with externality.

Several previous studies ([Schwartz and Repetto, 2000](#); [Williams, 2002](#); [Carbone and Smith, 2008](#)) investigate the impacts of a nonseparable utility function in externality. [Williams \(2002\)](#) shows that this non-separability could create a benefit-side tax-interaction effect.

²This result is analogous to the “over-consumption” in the literature on consumption externality ([Dupor and Liu, 2003](#); [Wendner and Goulder, 2008](#); [Eckerstorfer and Wendner, 2013](#)).

³In [Bovenberg and Goulder \(1996\)](#), the constraints involve either the inability to alter all tax rates or the inability to use revenues from environmental taxes in optimal ways. The policy constraints in this paper belong to the class of the inability to alter all tax rates.

Carbone and Smith (2008) calibrate a general equilibrium model with a nonseparable utility function in externality to the U.S. economy; they find that the tax-interaction effect is substantially affected by the non-separability. Households in all of these models are, however, homogeneous. Integrating the non-separability utility function and household heterogeneity allows us to, first, examine explicitly the role that an externality preference plays in determining consumption, income, and wealth inequalities and, second, link externality preference to the relationship between optimal taxation and consumption, income, and wealth inequalities. These two issues, though important in a policy circle, have not been addressed in previous formal models.

This paper also complements a series of influential papers by Fullerton and others (Fullerton and Heutel, 2007, 2010a; Fullerton et al., 2012; Fullerton and Monti, 2013) that study the incidences of externality taxes. This paper differs from the contributions in Fullerton and Heutel (2010a) and Fullerton et al. (2012) by integrating household heterogeneity. Fullerton and Monti (2013), integrating two types of households, study the distributional impacts of a pollution tax swap. While these models are concerned with the incidence of environmental policies, none is strictly an examination of policy effects on inequalities.

Our quantitative exercise, by considering a dynamic framework, adds to the literature on the optimality and the distributional impact of externality taxes in a general equilibrium setting (Bovenberg and Goulder, 1996; Fullerton and Heutel, 2007; Carbone and Smith, 2008; Fullerton and Monti, 2013; Rausch and Schwarz, 2016). Goulder (1995), Bovenberg and Goulder (1996), and Goulder et al. (1997) show that externality taxes could induce the so-called “tax interaction effect” with other pre-existing distortionary taxes such as income taxes. Any analysis, abstracting pre-existing distortionary taxes, may significantly bias the cost assessment of externality taxes. The literature (Fullerton and Metcalf, 1997; Parry and Bento, 2000; Bento and Jacobsen, 2007) on environmental policies proposes revenue-neutral environmental taxes that recycle the tax revenue from externality taxes to lower a pre-existing income tax, generating the so-called “double-dividend” (i.e., mitigating externality and the social cost of a pre-existing income tax). The pre-existing income tax therefore plays an important role in characterizing the optimal externality tax rate. While Hsu and Yang (2013) show that shutting down the tax distortion imposed on the consumption-saving decision will substantially affect the optimal income tax, works under a dynamic

framework are rare in the literature on optimal externality taxation. Our quantitative exercise extends the general equilibrium to a dynamic stochastic general equilibrium (DSGE) to remedy this deficiency.

In our DSGE model, the motives for redistributive taxation are for equity per se and social insurance as in [Mirrlees \(1971\)](#) and [Mirrlees \(1974\)](#), respectively. These two seminal works generate two lines of the optimal taxation literature.⁴ This paper contributes to these two strands of literature by considering the optimal taxation (i.e., the combination of the optimal externality and income taxes) that purposes to correct income distribution for equity, to provide social insurance against households' idiosyncratic risk, and to mitigate negative production externalities. The abstraction of the motive for either equity or social insurance may lower the optimal income tax. In either case, the introduction of an externality tax may underestimate the "tax interaction effect" and the "double-dividend". This paper contributes to this line of the literature on the front that the introduction of an externality tax combats externalities, and the tax revenue from this externality tax mitigates the social cost of a pre-existing income tax and serves as equity per se and social insurance.

This paper proceeds as follows. Section 2 introduces a stylized model in a general equilibrium. Section 3 extends the model to a dynamic and stochastic framework. Section 4 calibrates the parameter values of the model. Section 5 discusses the impact of the increasing concern over production externalities, and the policy constraints are examined in Section 6. Section 7 concludes the paper.

2 A Static Model

This section presents the simplest possible stylized model to shed light on the major mechanisms through which an income tax, an externality tax, and a concern over production externalities affect an economy in a general equilibrium. It analytically shows how the concern and the two taxes affect aggregate consumption, labor, and production externalities; moreover, it investigates their effects on the Gini coefficients of consumption and income, two critically important yet often neglected variables. This exercise is important because it

⁴Subsequent works on the former literature include [Strawczynski \(1998\)](#), [Low and Maldoom \(2004\)](#), and [Kanbur et al. \(2008\)](#). Readers who are interested in a recent study on the latter literature are referred to [Tuomala \(2010\)](#).

will help us understand the major mechanisms at play in the numerical analysis presented in the rest of the paper.

In this model economy, there is one period. The economy has three sectors: households, a production sector, and a government. There is a unit measure of utility-maximizing households. Households differ by their labor productivity l , where l follows a distribution $H(l)$, with \underline{l} and \bar{l} the lower and the upper supports of the distribution, respectively. In a production section, risk-neutral firms employ labor hours to produce final goods with a production externality. The government implements its policy instruments: the two taxes. There are two types of goods: labor hours and final goods. The final good is the numeraire.

2.1 Household

Each household is endowed with a unit of time, which is divided between labor n and leisure $1 - n$. Given a wage rate w , a government transfer T , and production externality E , each household chooses consumption c and leisure to maximize its utility. Following the literature on consumption externality, we embed the negative production externality into a utility function as follows:

$$U(l) = \max_{\{c,n\}} (c - \eta E)^\phi (1 - n)^{1-\phi},$$

where $\phi \in (0, 1)$ is a parameter denoting the relative importance of consumption versus leisure and $\eta \geq 0$ measures the importance of the disutility from production externality relative to consumption.⁵ A higher η reflects a higher disutility from externality. When $\eta = 0$, households are not concerned with production externality. Therefore, we can interpret η as the degree of concern over production externality relative to the consumption of a final good.

This functional form coheres with two empirical regularities. First, a higher level of production externality reduces household utility. Second, the marginal utility of production externality is given by $-\phi\eta\left(\frac{1-n}{c-\eta E}\right)^{1-\phi}$. It is well documented that the poor tend to consume less and work less. Hence, their marginal disutility of production externality is larger than

⁵We consider production externality in utility as in the literature on consumption externality in utility (Clark and Oswald, 1998; Dupor and Liu, 2003; Abel, 2005; Bowles and Park, 2005; Alonso-Carrera et al., 2008; Wendner and Goulder, 2008; Tideman et al., 2008; Wendner, 2010; Bishnu, 2013; Eckerstorfer and Wendner, 2013).

the rich. While the literature on environmental injustice provides evidence that traditionally disadvantaged groups, including the poor, tend to expose themselves more to each unit increase in pollution (Banzhaf and Walsh, 2008; Kaswan, 2008; Gamper-Rabindran and Timmins, 2011), a higher level of production externality may reduce household utility. Hence, the implied marginal disutility of externality does capture this feature.

The decisions on consumption and leisure are subject to a budget constraint. Each household receives a labor income wnl , where nl is an effective labor hour. Each household pays an income tax at a rate of $\tau \geq 0$ and receives a lump-sum transfer T from the government. The budget constraint is given by

$$c + (1 - \tau)wl(1 - n) = (1 - \tau)wl + T.$$

The optimal household consumption is given by the following to maximize the utility function under the budget constraint:

$$c^* = \phi[(1 - \tau)wl + T] + (1 - \phi)\eta E. \quad (1)$$

The income effect is standard: consumption strictly increases with income. Hence, c^* strictly increases with a wage rate w , a productivity level l , and a lump-sum transfer T . A higher income tax rate τ reduces income, thereby reducing consumption. It is well known that a household equates the marginal rate of substitution (MRS) (i.e., MU_{1-n}/MU_c , where MU_j is the marginal utility of j) to the relative price $(1 - \tau)wl$ at the optimum. Both higher η and E increase the marginal utility of consumption and thus reduces the MRS; therefore, a household consumes more and supplies more labor hours to increase the marginal utility of leisure in response to higher η and E .

At the optimum, a household labor supply is given by

$$n^* = \phi - (1 - \phi) \left(\frac{T - \eta E}{(1 - \tau)wl} \right). \quad (2)$$

More transfers generate a positive income effect: while leisure is considered a normal good, the optimal household labor supply n^* decreases with T . Both a higher wage and a higher productivity level increase returns to labor hours, inducing a substitution effect. Hence, n^* increases with w and l . A higher income tax rate, on the contrary, reduces returns to labor

hours and thus n^* .

It is straightforward to show that $n^* = 0$ if $l \leq \frac{1-\phi}{\phi(1-\tau)w}(T - \eta E)$ and $n^* = 1$ if $l \leq \frac{\eta E - T}{(1-\tau)w}$. Hence, a corner solution arises if $l \leq \frac{1-\phi}{\phi(1-\tau)w}(T - \eta E)$ or $l \leq \frac{\eta E - T}{(1-\tau)w}$. For ease of exposition, we assume that the lowest productivity level is sufficiently high throughout the section to restrict our attention to the interior solution (i.e., $n^* \in (0, 1)$). We will relax this assumption in our numerical exercise in Section 3.

2.2 Production Sector

The aggregate output of an economy is given by the following production function

$$Y = L^\gamma E^{1-\gamma}, \quad (3)$$

where $L \geq 0$ is the aggregate effective labor, $E \geq 0$ is a by-product of production, namely production externality, and $\gamma \in (0, 1)$ is a technological parameter of a production function.⁶ All markets are assumed to be perfectly competitive. Without loss of generality, we assume the existence of a representative firm operating the production. A firm is required to pay an externality tax $\nu \in (0, 1)$ per unit of externality.⁷

Given a wage rate w and an externality tax rate ν , a firm chooses L and E to maximize the profit:

$$\max_{L, E} Y - wL - \nu E.$$

The following first-order conditions are satisfied in the equilibrium for the interior solutions:

$$w = \gamma \frac{Y}{L}, \text{ and } \nu = (1 - \gamma) \frac{Y}{E}, \quad (4)$$

where the firm's choices of E and L equate the marginal cost to the corresponding marginal

⁶Copeland and Taylor (1994) show that emission, a by-product of production, could be modeled as an input under certain mild conditions. Thus, it is common to assume emission as an input in the production function in the literature (Fullerton and Heutel, 2010b; Tombe and Winter, 2015; Goulder et al., 2016; Rausch and Schwarz, 2016).

⁷Instead of assuming $\nu \in (0, 1)$, one can always normalize the unit of externality. See Tombe and Winter (2015).

product.

2.3 Government

The government redistributes tax revenues equally across households and, simultaneously, balances its budget so that

$$\underbrace{T}_{\text{Government Transfer}} = \underbrace{\int \tau w n l dH(l)}_{\text{Income Tax Revenue}} + \underbrace{\nu E}_{\text{Externality Tax Revenue}}. \quad (5)$$

The redistribution of income tax revenues to households corrects the income distribution for equity as in [Mirrlees \(1971\)](#). While the main purpose of the externality tax is to combat production externality, its tax revenue also benefits the economy for equity via a lump-sum transfer.⁸

2.4 General Equilibrium

This subsection mainly analyzes the properties of the general equilibrium, which is defined as follows. Conditional on an income tax rate τ and an externality tax rate ν , a general equilibrium is defined as the decision rule $(c^*(l), n^*(l))$ for households, aggregate consumption, externality, labor, the lump-sum transfer, and output $(C^*, E^*, L^*, T^*, Y^*)$, and the wage w^* such that

1. (Optimal Consumption and Leisure Levels): Given (w^*, T^*) , $(c^*(l), n^*(l))$ solves the household's optimization problem for all l , satisfying equations (1) and (2);
2. (Production of Goods): Y^* satisfies the production function (3);
3. (Profits Maximization): Given w^* , (L^*, E^*) maximizes the firm's profit, satisfying equations (4);
4. (Government Balanced Budget): T^* matches the total government expenditure, satisfying the budget constraint (5);
5. (All Markets Clearing): $L^* = \int n^*(l) l dH(l)$ and $Y^* = C^*$, where $C^* = \int c^*(l) dH(l)$.⁹

⁸Of course, there are many ways to spend the externality tax revenue; for example, they can be recycled to lower income taxes. It is a fruitful research area in designing the optimal way to spend the externality tax revenue.

⁹In the rest of this section, we omit $*$ but all the endogenized variables are meant in the general equilibrium.

The lump-sum transfer is given by the following using the government balanced budget condition and the labor market clearing condition:

$$T = \tau wL + \nu E, \quad (6)$$

which is independent of l . The aggregate labor supply is given by the following using the labor supply (2):

$$L = \phi \tilde{l} - (1 - \phi) \left(\frac{T - \eta E}{(1 - \tau)w} \right), \quad (7)$$

where $\tilde{l} \equiv \mathbb{E}_l(l)$, the first moment of $H(l)$. Using the firm's first-order conditions (4), the transfer (6), and the aggregate labor supply (7), it is straightforward to show that the aggregate labor supply can be further reduced to

$$L = \frac{\phi \gamma \nu (1 - \tau) \tilde{l}}{\gamma \nu (1 - \phi \tau) + (1 - \gamma)(1 - \phi)(\nu - \eta)}. \quad (8)$$

The derivation of equation (8) can be found in Appendix 8.1. The externality level is given by the following using the production function (3), the firm's first-order conditions (4), and the aggregate labor supply (8):

$$E = \left(\frac{1 - \gamma}{\nu} \right)^{\frac{1}{\gamma}} L = \left(\frac{1 - \gamma}{\nu} \right)^{\frac{1}{\gamma}} \frac{\phi \gamma \nu (1 - \tau) \tilde{l}}{\gamma \nu (1 - \phi \tau) + (1 - \phi)(1 - \gamma)(\nu - \eta)}. \quad (9)$$

Given L^* and E^* in (8) and (9), Y^* is uniquely solved using the production function (3). With E^* , L^* , and Y^* , the firm's first-order conditions (4) pin down the wage rate w^* as follows:

$$w = \gamma \left(\frac{1 - \gamma}{\nu} \right)^{\frac{1 - \gamma}{\gamma}}. \quad (10)$$

The aggregate labor supply L^* , the externality level E^* , and the wage rate w^* are uniquely pinned down in equations (8), (9), and (10), respectively. Hence, the unique T^* can be found using equation (6). The aggregate consumption level C^* is given by the following

using the household consumption schedule (1) and the transfer (6):

$$\begin{aligned}
C &\equiv \int_0^1 \phi \left[(1 - \tau)wl + T \right] + (1 - \phi)\eta E dH(l) \\
&= \phi \left[(1 - \tau)w\bar{l} + T \right] + (1 - \phi)\eta E \\
&= \phi \left[(1 - \tau)\tilde{l} + \tau L \right] w + \left[\phi\nu + (1 - \phi)\eta \right] E.
\end{aligned} \tag{11}$$

Since C^* strictly increases with w^* , E^* , and T^* and since the w^* , E^* , and T^* are all unique, C^* is unique. Using equations (1) and (2), it is straightforward to see that both c^* and n^* are unique for all l .

Proposition 1. *There exists a unique general equilibrium.*

Before proceeding to explore the impacts of η , τ , and ν on an economy, we discuss the measures of consumption and income inequalities. Consumption and labor income inequalities are, respectively, measured by the Gini coefficients of consumption c and the after-tax labor income $z \equiv (1 - \tau)wnl$, denoted by $G(c)$ and $G(z)$. The Gini coefficient for c is given by the following using the change-of-variable technique:

$$G(c) = \frac{1}{C} \int_{\underline{c}}^{\bar{c}} H(\xi)[1 - H(\xi)]dc,$$

where $\bar{c} \equiv \phi[(1 - \tau)w^*\bar{l} + T^*] + (1 - \phi)\eta E^*$, $\underline{c} \equiv \phi[(1 - \tau)w^*\underline{l} + T^*] + (1 - \phi)\eta E^*$, and $\xi \equiv [c - \phi(\tau w^*L^* + \nu E^*) - (1 - \phi)\eta E^*] / \phi(1 - \tau)w^*$. Using the integration by substitution, one can show that the Gini coefficient for c is given by

$$G(c) = \frac{\phi(1 - \tau)w}{C} \int_{\underline{l}}^{\bar{l}} H(x)[1 - H(x)]dx. \tag{12}$$

Since Gini coefficients are unaffected by any scale effect, $G(z) = G(nl)$. Using equation (2), one can show that $n^*l = \phi l - B$, where $B \equiv (1 - \phi)(T^* - \eta E^*) / (1 - \tau)w^*$. The Gini coefficient for z is given by the following using the change-of-variable technique:

$$G(z) = \frac{1}{L} \int_{n\underline{l}}^{n\bar{l}} H\left(\frac{nl + B}{\phi}\right) \left[1 - H\left(\frac{nl + B}{\phi}\right) \right] dnl,$$

where L^* appears in the Gini coefficient of z because $\int n(l)ldH(l) = L$. The Gini coeffi-

cient for z is given by the following using the integration by substitution:

$$G(z) = \frac{\phi}{L} \int_{\underline{l}}^{\bar{l}} H(x)[1 - H(x)]dx. \quad (13)$$

The Impacts of Externality Concerns. An increasing concern over production externality reduces the utility derived from consumption, thereby increasing the marginal utility of consumption and thus decreasing the marginal rate of substitution (i.e., MU_{1-n}/MU_c). Since households optimize their utility-maximizing problem by equating the marginal rate of substitution to a relative price between consumption and leisure, the decrease in the marginal rate of substitution alters the household's consumption and leisure schedules. Consequently, households respond by increasing their consumption and labor supply.

In an equilibrium, aggregate consumption, externality, labor, and output increase. The rise in the household consumption increases aggregate consumption and stimulates the demands for final goods. Firms respond by employing more labor hours, thereby increasing the demand for aggregate labor. On the other hand, the rise in the household labor supply increases the supply of aggregate labor. In the equilibrium, aggregate labor increases because of the increases in both the demand and supply of aggregate labor. With a higher level of aggregate labor, the marginal product of externality rises; firms respond by generating more output and externality. As a result, the increasing concern over a production externality increases aggregate consumption, externality, labor, and output.¹⁰

The response is heterogeneous across households: the labor supply of a more-productive household is less responsive to the rise in η for two reasons. First, a more-productive household tends to have more consumption and thus has less marginal utility of consumption. As discussed above, the rise in η will increase consumption because of the decrease in the marginal rate of substitution. The impact on more-productive households is smaller because they have less marginal utilities. Second, the marginal utility of leisure is larger for a more-productive household because it works more and takes up less leisure time. Hence, a slighter adjustment in the labor supply of a more-productive household allows it to reach new optimal consumption and leisure schedules. As a result, the increasing concern about externality increases a household labor supply disproportionately: the increase in the labor

¹⁰It is important to highlight that this result generally holds if the marginal utility of consumption increases with externality as in the literature on consumption externality.

supply of a more-productive household is smaller. In the end, the Gini coefficients of income and thus consumption decrease. The following proposition summarizes the impacts of externality concerns.

Proposition 2. *An increasing concern over production externality increases aggregate consumption, aggregate labor, aggregate output, and externality. It increases household labor supplies, and the effect is less pronounced for more-productive households. Furthermore, the increasing concern decreases the Gini coefficients of consumption and labor income.*

Proof. See Appendix 8.2. □

The Impacts of Externality Taxes. The literature has observed that a rise in an externality tax rate reduces externality because of the higher marginal cost of generating externality. This section adds to the literature by providing another channel. From the perspective of the firm, an increase in an externality tax rate reduces externality. The fall in externality reduces the marginal product of labor and thus the demands for labor. Meanwhile, the fall in externality, similar to the fall in concerns over production externality, increases the utility derived from consumption. Consequently, households will reduce their labor supplies. The fall in both the demand and the supply of labor decreases aggregate labor in the equilibrium. Such a decrease reduces the marginal product of externality and depresses the demands for externality. In the end, the externality drops further.

An increase in an externality tax rate reduces aggregate consumption and output. With the decreases in both aggregate labor and externality, aggregate output drops. The decrease in aggregate consumption, therefore, immediately follows in the equilibrium. Our model adds to the literature by providing one more channel. In fact, the reduction of externality, similar to the fall in the concern, reduces household consumption and labor supplies. Hence, the decrease in aggregate consumption is driven by the declines in both the demand and supply for final goods.

Similar to the response to changes in η , the effects of the increase in an externality tax rate is heterogeneous across households: the labor supply of a more-productive household is less responsive for the same reasons. Since a rise in an externality tax rate reduces externality, such a reduction decreases a household labor supply disproportionately: the decrease in the labor supply of a more-productive household is smaller. In the end, the

Gini coefficients of income and thus consumption increase. The following proposition summarizes the impacts of an externality tax.

Proposition 3. *An increase in an externality tax rate decreases aggregate consumption, aggregate labor, aggregate output, and externality. It decreases household labor supplies, and the effect is less pronounced for more-productive households. Furthermore, the increase in an externality tax rate increases the Gini coefficients of consumption and labor income.*

Proof. See Appendix 8.3. □

The Impacts of Income Taxes. Our model provides a new insight on the channel through which an income tax affects aggregate consumption and labor. A higher income tax rate reduces returns to labor hours. This standard substitution effect reduces household consumption and labor supplies.

Such a decline in a labor supply lowers aggregate labor in the equilibrium. Consequently, the marginal product of externality drops; firms respond by producing fewer final goods and generating a lower level of externality. Again, such a reduction in externality increases the utility derived from consumption and stimulates a series of household responses: further declines in both household labor supply and consumption. This general equilibrium reinforces the effects of an income tax on the declines in aggregate consumption, aggregate labor, aggregate output, and externality. We skip the intuition of an income tax on income inequality because it resembles the mechanisms in Propositions 2 and 3. The following proposition summarizes the impacts on an income tax.

Proposition 4. *An increase in an income tax rate decreases aggregate consumption, aggregate labor, aggregate output, and externality. It decreases household labor supplies and the effect is less pronounced for more-productive households. Furthermore, the increase in an income tax rate increases the Gini coefficient of labor income.*

Proof. See Appendix 8.4. □

In summary, this section shows that externality increases as the public's concern with production externality increases. Moreover, such an increasing concern is shown to improve consumption and income inequality measured in terms of a Gini coefficient. This

section also uncovers an additional channel through which increases in externality and income tax rates depress externality. Furthermore, an externality tax is shown to increase consumption and income inequalities.

This analysis extends earlier analytical works on externality taxes in a general equilibrium by considering household heterogeneities and production externality in utility. These two features allow us to examine the impacts of concerns over production externality and the two taxes on other important variables, including aggregate consumption, aggregate labor, and the level of externality. More importantly, we can analytically examine how and why the impacts on household labor supply differ across households. This examination enables us to investigate the impacts of the two taxes on the Gini coefficients of consumption and income, two critically important yet often neglected variables.

Moreover, the rationales of this simple model provide clear explanations as to how the concerns and the two taxes affect the equilibrium variables not only in a static model but also in a more “realistic” framework in the next section. Our numerical analysis shows that all the features in the three propositions (Propositions 2-4) preserve in a dynamic framework and shows the empirical significance of these features.

3 A Dynamic Model

To study the impacts of externality and income taxes on income and wealth distributions, this section extends a general equilibrium to a DSGE model. The static model in the previous section differs from a DSGE framework in at least two dimensions. First, a static model abstracts the decision on savings. Nevertheless, both an income tax and an externality tax affect the tradeoffs between consumption and labor supply and between current and future consumptions. While the former decision mainly affects labor incomes, the latter decision changes capital incomes. In a dynamic framework, the two decisions influence the accumulation of wealth. Unlike a static model, a DSGE model allows us to account for the impacts of the two taxes on wealth and its distribution and, more importantly, the cost of the dynamic distortion via the accumulation of wealth.

Second, income/externality taxes and transfers provide additional benefit to an economy in a stochastic framework. In a static model, an income tax could play its major role in

equity and an externality tax mainly combats production externality. With an idiosyncratic productivity shock to an imperfectly insured market, tax revenues and transfers serve as a social insurance as partial precautionary savings in addition to its role in equity.

This section constructs a heterogeneous agent model that closely follows the seminal work of [Aiyagari \(1994\)](#), in which households face idiosyncratic earnings uncertainty that cannot be insured. This model provides a framework to study endogenous cross-sectional consumption, income, and wealth distributions that largely depend on policy instruments. Our model also features, in addition to the decision on savings as in [Aiyagari \(1994\)](#), the choice between labor and leisure, which is shown to be important in the prescription of optimal tax codes ([Hsu and Yang, 2013](#)). Furthermore, negative production externality (emission) is embedded into a utility function so that policymakers are concerned with externality not because externality hurts a production sector, but because households are affected by production externality, providing policymakers incentives to combat externality using the two policy tools: externality and income taxes.

In this model economy, time is discrete and is denoted by $t = \{0, 1, \dots, \infty\}$. There are four sectors in the economy: households, a production sector, an abatement sector, and a government. There is a unit measure of utility-maximizing households that are infinitely lived and risk-averse. Households differ by their histories of realizations of idiosyncratic shocks to their labor productivity. In a production sector, a risk-neutral firm employs aggregate capital and labor to produce goods with externality. The government implements its policy instruments to maximize the welfare of an economy. We will discuss the measure of welfare after outlining the basic model setup. In response to these policies, firms allocate a fraction of resources to abate externality. There are four types of goods: labor hours, capital, final goods, and abatement services. The final good is the numeraire.

3.1 Labor Productivity Shock

Labor productivity is subject to idiosyncratic shocks. At time 0, all households begin with the same productivity level l_0 . At the beginning of each period t , labor productivity l_t is realized. To simplify the analysis, this stochastic process of the shock is assumed to be identical and independent across households and is assumed to follow a Markov chain with a stationary transition over time. We normalize the mean productivity l_t to be unity. The

effective labor hour equals $n_t l_t$, where n_t is the number of labor hours. The market price of a unit of effective labor hours is w_t .

3.2 Production Sector and Abatement Sector

The aggregate output of an economy is given by the following production function

$$Y_t = \mu_t K_t^\alpha L_t^{1-\alpha}, \quad (14)$$

where $K_t \geq 0$ and $L_t \geq 0$ are aggregate capital and effective labor, respectively, and $\alpha > 0$ is a technological parameter of a production function. A firm allocates a fraction of resources $\mu_t \in [0, 1]$ to a production sector and allocates the rest to abate production externality. This externality is a by-product of production and is given by

$$E_t = \mu_t^{\frac{1}{1-\gamma}} K_t^\alpha L_t^{1-\alpha}, \quad (15)$$

where $\gamma > 0$ is a technological parameter of an abatement production function. It is straightforward to see that $E_t \geq 0$ increases with μ_t : the smaller the fraction of resources (i.e., $1 - \mu_t$) allocated to an abatement sector, the greater the negative production externality.

A firm is required to pay an externality tax $\nu \geq 0$ per unit of externality. Of course, a firm has an option to allocate a fraction of resources to abate externality. Without abatement, μ_t is one, and the externality reaches its maximum level $K_t^\alpha L_t^{1-\alpha}$. As more resources are allocated to abatement (i.e., μ_t drops), an externality falls. Using the two production functions (14) and (15), we have

$$Y_t = (K_t^\alpha L_t^{1-\alpha})^\gamma E_t^{1-\gamma}, \quad (16)$$

which is a constant return to scale technology. All markets are assumed to be perfectly competitive. We assume the existence of a representative firm operating the production without loss of generality.

Given an hourly wage w_t , an interest rate r_t , and an externality tax rate ν , a firm, instead

of choosing K_t , L_t , and μ_t , chooses K_t , L_t , and E_t to maximize its profit:

$$\max_{E_t, K_t, L_t} Y_t - w_t L_t - r_t K_t - \nu E_t.$$

The following first-order conditions are satisfied in the equilibrium for an interior solution:

$$r_t = \gamma \alpha \frac{Y_t}{K_t}, w_t = \gamma(1 - \alpha) \frac{Y_t}{L_t}, \text{ and } \nu = (1 - \gamma) \frac{Y_t}{E_t}, \quad (17)$$

An externality tax payment is the only cost of externality for a firm. The third equation equates the marginal benefit of abatement to its marginal cost. To abate a unit of externality, a firm saves a unit of externality tax and simultaneously reduces its production level by the marginal product of externality. One can show that if $\nu < 1 - \gamma$, the marginal cost of abatement exceeds its marginal benefit for all $\mu_t \in [0, 1]$. A unique corner solution arises: a firm allocates no resources to an abatement sector (*i.e.*, $\mu^* = 1$), and the corresponding externality level is given by $E_t^* = K_t^{*\alpha} L_t^{*1-\alpha}$. In this case, only the first two equations of the first-order conditions (17) hold to determine the optimal K_t and L_t . In particular, if an externality tax is zero (so that $\nu < 1 - \gamma$), this firm problem reduces to the standard one with the production function $Y_t = K_t^\alpha L_t^{1-\alpha}$ and choice variables K_t and L_t . Hence, an economy without any externality tax is a special case of our model and serves as a baseline model later in this paper.

3.3 Household

Given a wage rate w_t , an interest rate r_t , a government transfer T_t , and an externality level E_t , each household chooses the sequence of consumption and leisure to maximize its expected discounted lifetime utility:

$$\max_{\{c_t, n_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, E_t).$$

And the choices of c_t and n_t are subject to the following budget and borrowing constraints:

$$\begin{aligned} k_{t+1} &= k_t + (1 - \tau)(w_t n_t l_t + (r_t - \delta)k_t) - c_t + T_t, \text{ for all } t = 0, 1, 2, \dots \\ k_{t+1} &\geq b, \text{ for all } t = 0, 1, 2, \dots \\ c_t &\geq 0, \text{ and } 0 \leq n_t \leq 1, \text{ for all } t = 0, 1, 2, \dots \end{aligned}$$

where $\beta \in (0, 1)$ is a time discount factor, $\delta \geq 0$ is a depreciation rate, and $\eta \in [0, 1]$ measures the importance of externality relative to consumption. Each household receives a labor income $w_t n_t l_t$ and a capital income $(r_t - \delta)k_t$, where $n_t l_t$ is an effective labor hour, and $r_t - \delta$ is the return rate to capital. Each household pays an income tax from the two incomes at a rate of $\tau \geq 0$.¹¹ Besides, each household receives a transfer T_t from the government in each period. Households face a borrowing constraint b , which serves two purposes. First, it rules out Ponzi schemes, and second, it provides a motive for precautionary savings to against future income fluctuations.

Following the literature on consumption externalities (Clark and Oswald, 1998; Bowles and Park, 2005; Tideman et al., 2008), we embed negative production externality into a utility function as follows:

$$u(c_t, n_t, E_t) = \frac{[(c_t - \eta E_t)^\phi (1 - n_t)^{1-\phi}]^{1-\sigma}}{1 - \sigma},$$

where c_t and n_t are consumption and labor hours at time t , and σ is a parameter governing the degree of risk aversion of a household. Such a household maximization problem can be shown to be satisfied by a Bellman equation at any t as follows:

$$V(k_t, l_t) = \max_{c_t, n_t} u(c_t, n_t, E_t) + \beta \mathbb{E}_t V(k_{t+1}, l_{t+1}), \quad (18)$$

subject to the household budget and borrowing constraints, where $V(k_t, l_t)$ denotes the value function of households of state $s_t \equiv (k_t, l_t)$. We denote by $F_t(\cdot)$ the cumulative distribution function of s_t . The heterogeneity across households arises from a labor productivity shock, which in turn affects household decisions on their consumption and labor supply and thus income and wealth distributions. The household expected discounted life-

¹¹We follow Hsu and Yang (2013) to assume that the tax rate does not distinguish sources of income in general, which is similar to the case in the current U.S. personal income tax code.

time utility, denoted by $V^*(s_0)$, is the optimal value function given by the Bellman equation (18).

3.4 Government

We follow [Conesa et al. \(2009\)](#) and [Hsu and Yang \(2013\)](#) to assume that the government redistributes tax revenues equally across households and simultaneously balances its budget in each period so that

$$\underbrace{\int T_t(s) dF_t(s)}_{\text{Government Transfer}} = \underbrace{\nu E_t}_{\text{Externality Tax Revenue}} + \underbrace{\int \tau(w_t l_t n_t(s) + r_t k_t) dF_t(s)}_{\text{Income Tax Revenue}}. \quad (19)$$

The main purpose of the externality tax is to combat production externality. Here, the externality tax revenue could also serve as social insurance to absorb income fluctuations. Similar to [Hsu and Yang \(2013\)](#), the redistribution of income tax revenues to households in a lump-sum manner has two benefits. First, the lump-sum transfer serves as social insurance to absorb income fluctuations and share the idiosyncratic risk across households as in [Mirrlees \(1974\)](#) and [Varian \(1980\)](#). Second, the transfer corrects the income distribution for equity as in [Mirrlees \(1971\)](#).

3.5 Stationary Equilibrium

Our analysis focuses on a stationary equilibrium because it allows us to focus on the effects of the two taxes in the long-run.¹² Conditional on an income tax τ and an externality tax ν , a stationary equilibrium is defined as aggregate output level Y^* , aggregate consumption C^* , aggregate capital K^* , aggregate labor L^* , externality E^* , investment I^* , the lump-sum transfer T^* , the decision rule $(c^*(s), n^*(s), k'^*(s))$ for households, the state distribution $F^*(s)$, and the relative prices of capital and labor (r^*, w^*) such that the following is satisfied.

1. (Production of Goods): Y^* satisfies the production function (16).

¹²Readers who are interested in the short-run effects of environmental policy instruments are referred to [Fischer and Springborn \(2011\)](#), [Heutel \(2012\)](#), [Annicchiarico and Di Dio \(2015\)](#), and [Dissou and Karnizova \(2016\)](#).

2. (Profits Maximization): Given (r^*, w^*, T^*) , (K^*, L^*, E^*) maximizes firm's profit, satisfying equations (17).
3. (Optimal Consumption and Leisure Level): Given (r^*, w^*) , $(c^*(s), n^*(s), k'^*(s))$ solves the household's maximization problem (18) for all s .
4. (Government Budget Balanced): The government balances its budget, satisfying the budget constraint (19).
5. (Stationary Type Distribution): Given (r^*, w^*) , $F^*(s)$ is consistent with the decision rule $k'^*(s)$ and the Markov chain for all labor productivity shocks l .
6. (All Markets Clearing): $K^* = \int k^*(s)dF^*(s)$, $L^* = \int n^*(s)ldF^*(s)$, $Y^* = C^* + I^*$, and $I^* = K'^* - (1 - \delta)K^*$, where $C^* = \int c^*(s)dF^*(s)$.

In the stationary equilibrium, the state distribution is unchanged over time, implying that aggregate capital and labor are also constant. It is worth noting that the state facing each household is time-variant because there is an idiosyncratic shock. Since the analytical results of DSGE models with heterogeneous households are allowed in very few exceptions, we will analyze the equilibrium numerically.

3.6 Welfare Criterion

We assume a utilitarian welfare criterion, which is the average of the expected lifetime values of all households in the stationary equilibrium:

$$SW = \int_0^1 V^*(s)dF^*(s). \quad (20)$$

The government chooses the policy tools $\{\nu^*, \tau^*\}$ to maximize this welfare function (20) subject to the budget constraint (19) and policy constraints (if any).

4 Parameterization

Our model is calibrated to the U.S. annual data. We closely follow the calibration strategy in Aiyagari (1994) and Hsu and Yang (2013). As noted by Aiyagari (1994), parameter values for preference and technology are quite consistent in the postwar U.S. economy, and the parameter values in this paper are commonly adopted in macroeconomic models.

Following [Aiyagari \(1994\)](#) and [Hsu and Yang \(2013\)](#), α is set to 0.36, and the depreciation rate δ is set to 0.08. We follow [Hsu and Yang \(2013\)](#) to set the elasticity of utility with respect to labor hours ϕ to 0.4 and the risk aversion σ to 2, both of which are quite standard in the literature. We will show in subsection 6.4 that the key results of this paper are not sensitive to the degree of risk aversion. The discount factor β is 0.96 as in [Aiyagari \(1994\)](#) and [Hsu and Yang \(2013\)](#). Following [Aiyagari \(1994\)](#), [Huggett \(1997\)](#), and [Hsu and Yang \(2013\)](#), the borrowing constraint b is set to zero. We assume that production externality is generated from pollution, which is proportional to the energy expenditure in a production sector. Hence, γ is set to 0.02 so that the capital, labor, and energy shares of output are 0.3528, 0.6272, and 0.02, respectively.

We lack the literature that calibrates the parameter value of η . Thus, we consider a wide range of η from 0 to 0.2. We first examine an economy in which households are not concerned with negative production externality (i.e., $\eta = 0$). This will serve as a baseline model. We believe $\eta \in (0, 0.2)$ because $\eta = 0.2$ means a unit increase of externality will reduce the consumption of the final goods in terms of the household's utility values by 0.2 units. Hence, the exploration for $\eta \in [0, 0.2]$ should include the U.S. economy during the entire transition of the increasing concerns over production externality. Table 1 summarizes the parametric values used in our model.

We follow [Hsu and Yang \(2013\)](#) to calibrate the labor endowment shock l . This shock, the idiosyncratic component in our model, is assumed to follow a Markov process with five possible states: $l \in \{0.3424, 0.5852, 1, 1.7089, 2.9202\}$ with the associated transition matrix:

$$Pr(l'|l) = \begin{pmatrix} 0.8072 & 0.1925 & 0.0003 & 0 & 0 \\ 0.0694 & 0.7886 & 0.1418 & 0.0001 & 0 \\ 0.0001 & 0.1010 & 0.7980 & 0.1010 & 0.0001 \\ 0.0000 & 0.0001 & 0.1418 & 0.7886 & 0.0694 \\ 0 & 0 & 0.0003 & 0.1925 & 0.8072 \end{pmatrix}.$$

The associated stationary distribution is unique and is given by

$$\phi(l) = (0.0874, 0.2424, 0.3405, 0.2424, 0.0874)$$

Table 1: Summary of Parameters

Parameter	Description	Value	Source
σ	Risk Aversion	2	Aiyagari (1994) & Hsu and Yang (2013)
β	Discount Factor	0.96	Aiyagari (1994) & Hsu and Yang (2013)
ϕ	Elasticity of Utility	0.4	Hsu and Yang (2013)
η	Externality Weight in Utility	[0,0.2]	
b	Borrowing Constraint	0	Aiyagari (1994) & Huggett (1997)
α	Capital Share	0.36	Aiyagari (1994) & Hsu and Yang (2013)
γ	Non-Externality Share	0.98	
δ	Depreciation Rate	0.08	Aiyagari (1994) & Hsu and Yang (2013)

For technical details of the calibration of this idiosyncratic component, readers are referred to [Hsu and Yang \(2013\)](#).

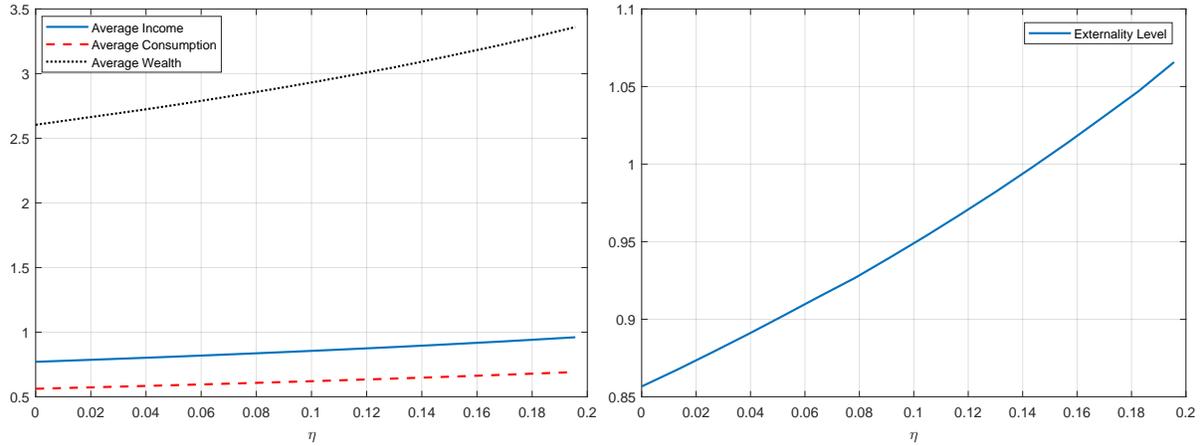
5 Increasing Concerns Over Production Externalities

This section investigates how an increasing concern over production externality affects consumption, income, wealth, and their distributions. We begin our analysis with a baseline model in which households are not concerned with any production externality, and there is no externality tax. In this model economy, an income tax revenue is the only source to finance a lump-sum transfer.

The optimal income tax rate is about 10.55 percent. On the cost side, an income tax distorts decisions between labor and leisure and between current and future consumptions. Meanwhile, an economy benefits from the income redistribution due to equity and social insurance. While the marginal utility of consumption is higher for the poor, the income redistribution from the rich to the poor (i.e., the equity) increases the welfare based on a utilitarian welfare criterion. Also, the income redistribution could serve as precautionary savings (e.g. social insurance) against idiosyncratic shocks to smooth household consumption. The role as social insurance increases welfare in a market that cannot be fully-insured. At the optimal income tax, the marginal cost of distortion equals its marginal social benefit.

Since [Hsu and Yang \(2013\)](#) analyzed a similar economy, interested readers are referred to their paper. Throughout this section, the income tax rate is set at 10.55 percent.

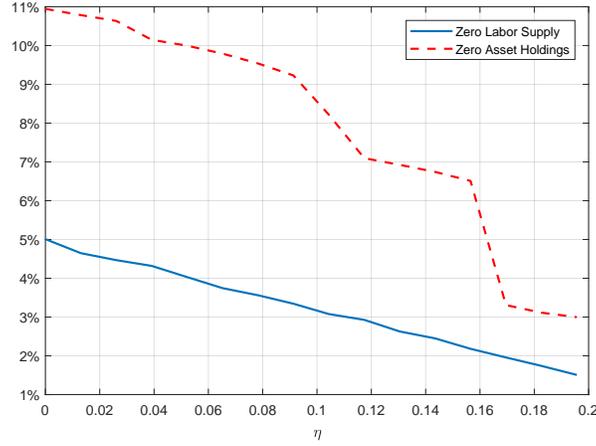
Figure 1: Impacts of Increasing Concerns on Consumption, Income, Wealth, and Externality



We now proceed to analyze the economic consequences of the increasing concern. Figure 1 presents aggregate consumption, income, and wealth as the concern over production externality η varies. The figure shows that aggregate consumption, income, and wealth increase with the concern. Intuitively, an increasing concern reduces the utility derived from household consumption, causing the marginal utility of consumption to rise. This induces household consumption and labor supply to rise in the equilibrium (see Proposition 2). The increases in labor supply and thus labor income accumulate wealth. Consequently, both aggregate labor and capital supplies increase to sustain a higher level of aggregate consumption in the stationary equilibrium. Aggregate consumption, income, and wealth increase with this preference parameter for the externality η , as predicted by Proposition 2.

Figure 1 shows that an externality level increases with η . As explained, in response to an increasing concern, households demand more final goods; firms respond by producing more goods and generating a higher level of production externality. While the increased consumption coheres with the overconsumption in the literature on consumption externalities ([Dupor and Liu, 2003](#); [Liu and Turnovsky, 2005](#)), this paper complements the literature in the context of this added externality effect. Such an added externality effect shares the same notion in ([Chang et al., 2018](#)), which shows that the promotion of environmentally-friendly products could increase pollution.

Figure 2: Impacts of η on the Fractions of Households Not Working or Holding Asset



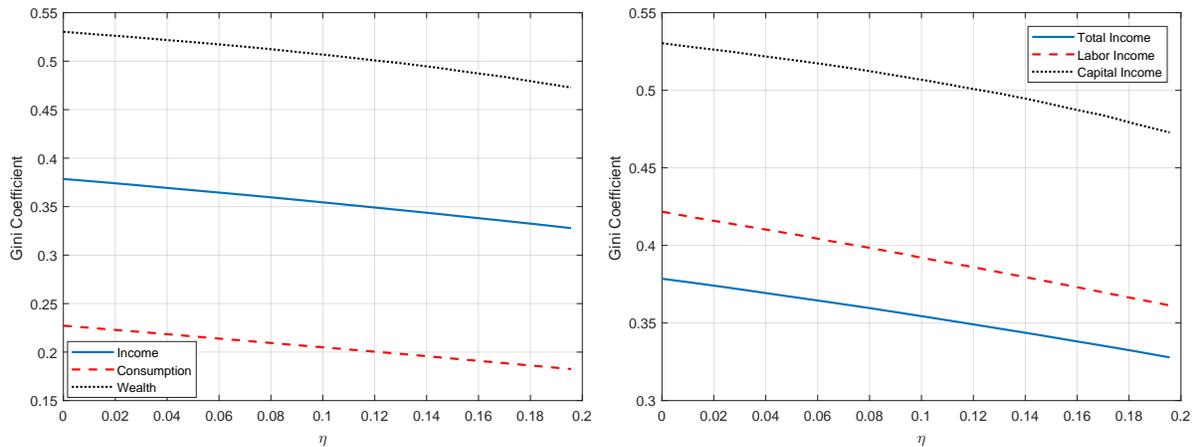
The fractions of households supplying zero labor and those holding zero wealth decrease with the concern. Figure 2 illustrates the two fractions against η . It has been known that optimal income taxes are compatible with the equilibrium, in which non-trivial fractions of households do not work or accumulate wealth (Kaplow, 2011; Hsu and Yang, 2013). We show that it is also compatible in the presence of production externality; moreover, these two fractions are shown to fall as the public becomes more concerned with externality.

This increasing concern makes resource distributions more equal. Figure 3 illustrates the Gini coefficients of consumption, income, and wealth as the concern varies. These coefficients cohere with the empirical order as in Aiyagari (1994): there is less dispersion in consumption and greater dispersion in wealth compared with income.

As shown in Proposition 2, the Gini coefficients of consumption and income decrease with η . In response to an increasing concern, households increase their labor supplies disproportionately: the increase is less pronounced for a more-productive household. With a higher level of a household consumption, a more-productive household is less affected by the increasing concern because the diminishing marginal utility of consumption ensures the impacts of the increasing concern on its marginal utility of consumption are smaller. Meanwhile, a more-productive household takes less leisure than her less-productive counterparts; hence, the diminishing marginal utility of leisure requires a smaller adjustment of a marginal utility of leisure in a more-productive household. These two reasons ex-

plain why the distribution of labor income in terms of the Gini coefficient improves, in line with Proposition 2. A similar mechanism can be applied: the accumulation of capital becomes slower for a wealthier household, causing the Gini coefficient of capital income to decline. Since the distributions of the labor and capital incomes become more equal, the Gini coefficients of income, wealth, and, thus, consumption decrease. It is worth noting that the decision on savings is abstracted from a static model; we were unable to examine the impacts of the increasing concern on the distributions of capital income and thus wealth. This numerical analysis adds to the literature demonstrating that the inequalities of consumption, income, and wealth become less severe as the public is more concerned with production externality.

Figure 3: Impacts of η on the Gini Coefficients of Consumption, Income, and Wealth



6 Optimal Taxation

This section investigates the interplay of the optimal externality and income taxes and explores how policy constraints affect the optimal taxation and their consequences. This analysis is informative for two reasons. First, it provides an opportunity to measure aggregate consumption, income, wealth, and their distributions at the optimum under realistic policy constraints. In addition to aggregate welfare, policymakers are, undoubtedly, interested in the comparisons of these economic outcomes, especially their distributions, for political considerations. Second, this exercise enhances our understanding of the functioning of

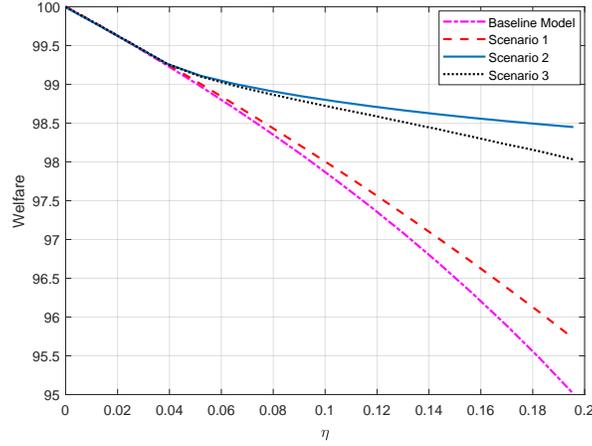
these taxes in combating negative production externality and in redistributing resources for equity and social insurance.

In particular, we analyze three case scenarios (policy constraints). First, only income taxes are adjusted in the absence of an externality tax. That is, $\{\nu, \tau\} \subseteq \{0, \mathbb{R}_+\}$. This case seems extreme but appears at the federal level in the United States. Second, both externality and income taxes are freely adjusted simultaneously (i.e., $\{\nu, \tau\} \subseteq \mathbb{R}_+^2$). In other words, there exists no policy constraint. This is an ideal case that the literature typically analyzes. Third, externality taxes are adjusted at the optimal income tax rate chosen in the first case. Often, income tax rates were designed without expecting or considering that an externality tax is (allowed) to be imposed. Once an externality tax is introduced, there may exist another policy constraint in refining the pre-existing income tax. Hence, $\{\nu, \tau\} \subseteq \{\mathbb{R}_+, \tau_1^*\}$, where τ_j^* is the optimal income tax rate in the case scenario $j \in \{1, 2, 3\}$.

Figure 4 displays social welfare in the baseline model and the three case scenarios. We normalize social welfare in terms of the utilitarian welfare criterion (20) in the baseline model, in which the public is not concerned with production externality, to 100. Two points emerge from the figure. First, welfare is the highest in the second case because there exists no policy constraint. Next in line is the third case scenario, followed by the first one. Since both the tax rates ($\nu = 0$ and $\tau = 0.1055$) are fixed for all η in the baseline model, an economy under this policy constraint has the lowest social welfare. Second, social welfare declines with η regardless of the policy constraint. When households dislike negative externality more, social welfare falls with η . In addition, as shown in Proposition 2, there are more externalities if the public becomes more concerned with externality, further decreasing social welfare as η increases.

Figure 5 illustrates the optimal income and externality tax rates in the three case scenarios. In response to an increasing concern over externality, the optimal income tax increases when the adjustment of an externality tax is not permitted (Scenario 1). As shown in Propositions 3 and 4, both taxes could reduce externality; hence, the optimal income tax increases to combat externality to increase social welfare. However, given this optimal income tax rate, the government will still prefer to increase an externality tax rate (if it is allowed to be adjusted as in Scenario 3). This significant increase in the optimal externality tax rate in Scenario 3 reflects that it is more efficient to adjust externality tax, not income tax, to

Figure 4: Welfare under Policy Constraints at Optimum



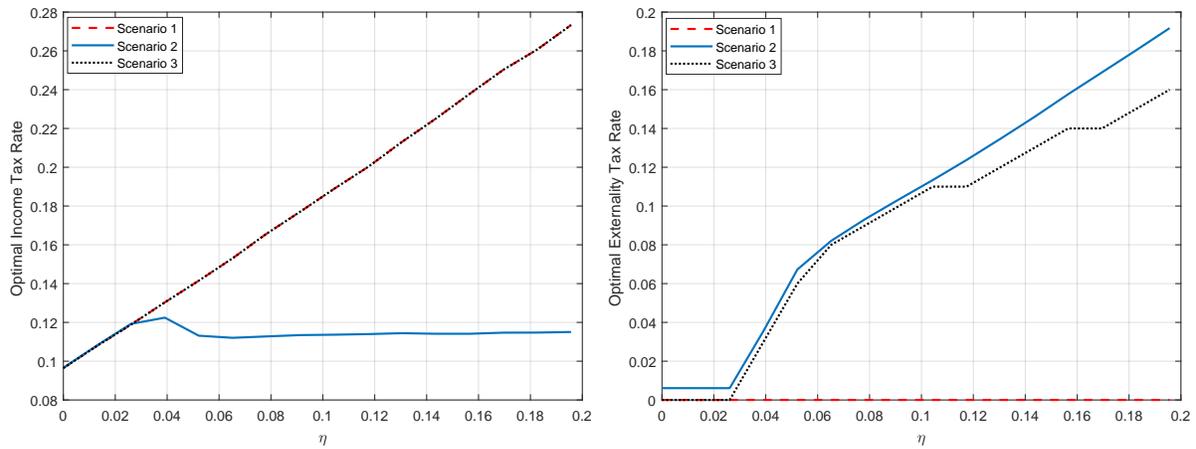
combat externality.

The responses of the two taxes are different in Scenario 2 in which the two taxes are freely chosen. Figure 5 shows that when η is below 0.04, the optimal income tax slightly increases with η . It remains unaffected for all $\eta \in (0.05, 0.20)$. In contrast, the optimal externality tax increases over 19 times from $\eta = 0$ to $\eta = 0.2$. This result coheres with that in Scenario 3: the adjustment of an externality tax rate rather than an income tax rate is more efficient in combating externality. The literature (Goulder, 1995; Bovenberg and Goulder, 1996; Goulder et al., 1997) suggests that the introduction of externality taxes will exaggerate the distortionary cost of the pre-existing income tax, and the size of this exaggeration increases with the income tax. Hence, this exaggeration is more pronounced in Scenario 3 than 2 because τ_3^* is far above τ_2^* . The social benefit of an externality tax to combat externality is so large that even though the externality tax could enlarge the distortionary cost of the income tax much more in Scenario 3, ν_3^* is just slightly below than ν_2^* , where ν_j^* is the optimal externality tax in the case scenario $j \in \{1, 2, 3\}$.

In brief, there is little room for improvement in social welfare if an externality tax is not allowed to be adjusted. In Figure 4, social welfare is close in magnitude in the baseline model and in Scenario 1, in which an externality tax rate cannot be adjusted. Once an externality tax is freely chosen as in Scenarios 2 and 3, welfare improves significantly, and the improvement is, as expected, more pronounced when the public becomes more concerned with externality. It is worth noting that such an improvement does not arise from

the interaction between an income tax and an externality tax. Notice that the two taxes are freely adjusted in Scenario 2 and, to a certain extent, are chosen in Scenario 3. According to Figure 5, the optimal income tax τ_2^* hardly reacts to the change in the preference parameter η : the adjustment in the optimal externality tax alone improves welfare significantly. We will compare the impacts on aggregate consumption, income, wealth, and their distributions under various policy constraints and discuss the intuitions behind these impacts of the taxes in detail.

Figure 5: The Optimal Taxation



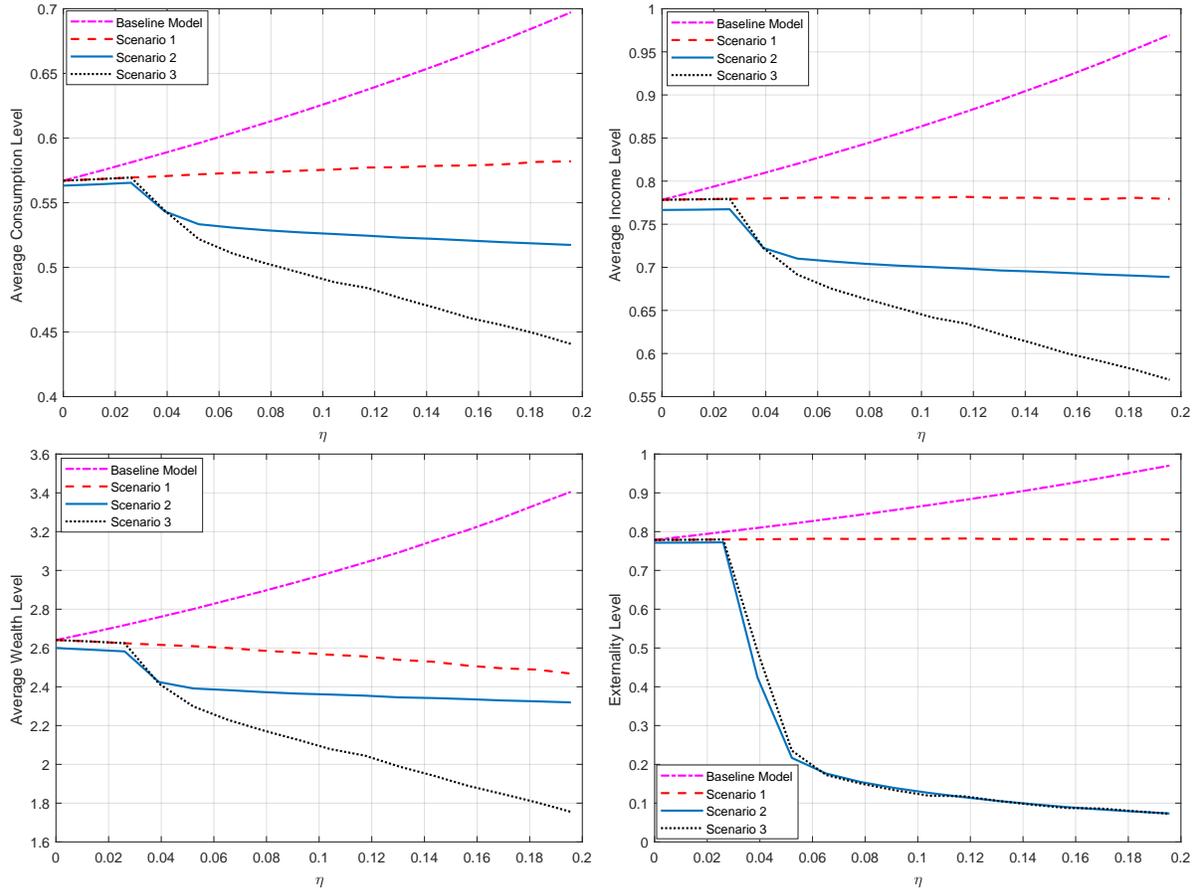
6.1 Case Scenario 1: $\{\nu, \tau\} \subseteq \{0, \mathbb{R}_+\}$

In response to an increasing concern about externality (i.e., the increase in η), the optimal income tax increases as shown in Figure 5. In the absence of an externality tax, an income tax is the only policy instrument. The difference between this case and the baseline model is solely driven by the higher income tax, and the comparison between the economies in this case and the baseline model illustrates the impacts of a higher income tax on economic activities.

Apparently, an increase in the τ reduces the returns to labor and capital, slowing the accumulation of wealth and thus generating fewer goods and production externalities in a stationary equilibrium. This policy is costly: as illustrated in Figure 6, it reduces aggregate consumption, income, and wealth. Meanwhile, an increase in the τ generates more tax

revenues for redistribution and precautionary saving, both of which increase social welfare. As highlighted above, the optimal income tax equates the marginal social benefit to its marginal cost of distortion.

Figure 6: Consumption, Income, Wealth, and Externality under Policy Constraints at Optimum

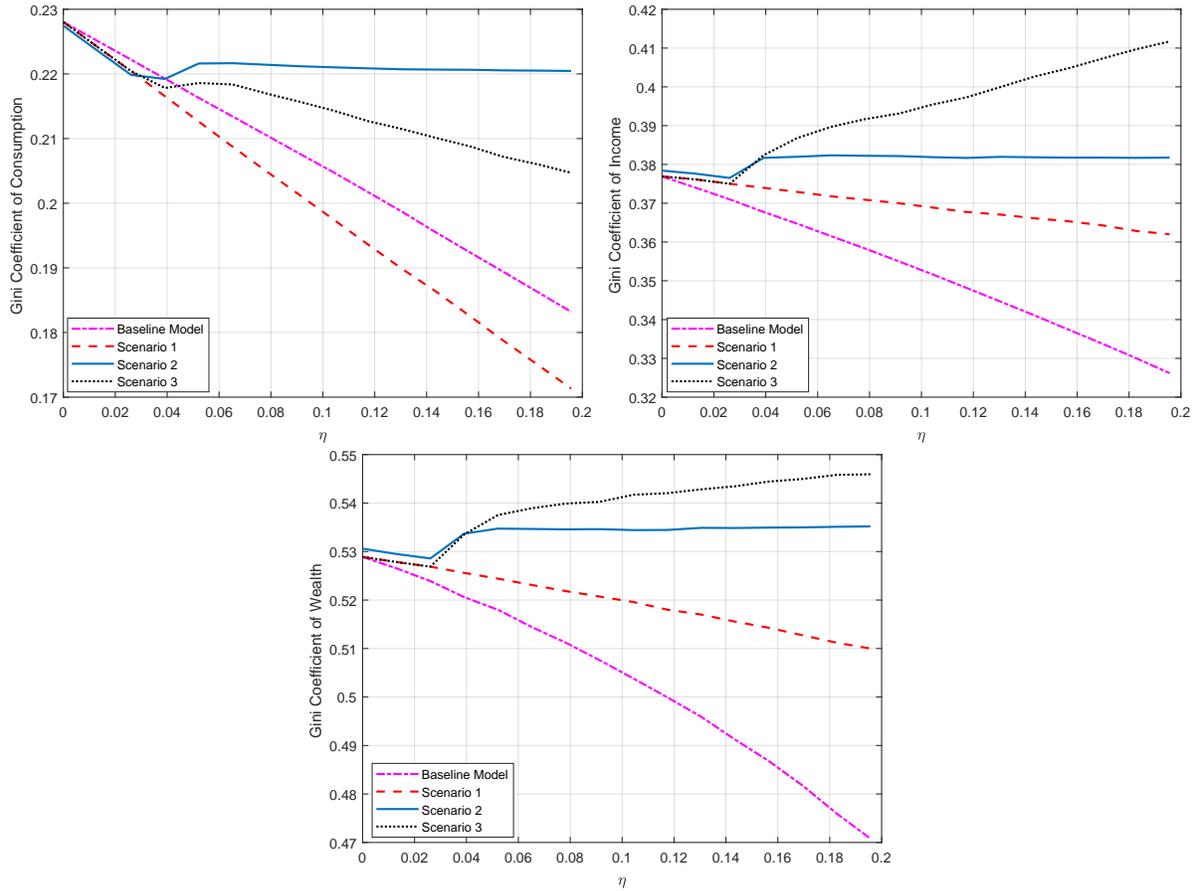


Households respond to this policy differently. Figure 7 displays the Gini coefficients of consumption, income, and wealth as η varies. Two points merge from this figure. First, the distributions of income and wealth, in terms of Gini coefficients, are more disperse relative to the baseline model. As shown in Proposition 4, household labor supply is less sensitive to policy changes in a more-productive household. When an income tax increases, there is a smaller proportional reduction of labor income in a more-productive household. Consequently, the Gini coefficient of labor income increases. A similar mechanism can be applied to capital income, causing the Gini coefficient of capital income to rise. Hence, the optimal taxation widens the income and wealth inequalities, measured in the Gini coefficients, in

Scenario 1.

Second, the optimal taxation improves consumption inequality in Scenario 1, relative to the baseline model. A higher income tax in Scenario 1 generates more tax revenues and thus transfer amounts. The transfer in a lump-sum fashion favors the poor because of a larger proportional increase of their income. Therefore, the Gini coefficient of consumption is lower in Scenario 1.

Figure 7: Consumption, Income, and Wealth Inequalities under Policy Constraints at Optimum



6.2 Case Scenario 2: $\{\nu, \tau\} \subseteq \mathbb{R}_+^2$

In the absence of policy constraints, the optimal income tax hardly reacts to changes in the preference parameter η , but the optimal externality tax rate tends to increase with η . Hence, the differences between Scenario 2 and the baseline model is largely attributed

to the higher externality tax in Scenario 2. Figure 6 shows that a higher externality tax generates lower consumption, income, and wealth and a lower externality level relative to the baseline model. The Gini coefficients of consumption, income, and wealth are higher in Scenario 2 as shown in Figure 7. These results are, to a certain extent, expected because of Proposition 3.

This numerical exercise adds to the literature by comparing the consequences of the above variables between Scenarios 1 and 2. According to Figure 6, aggregate consumption, income, and wealth levels are close in the two scenarios for all η . However, the externality level is substantially lower at a higher externality tax level, largely explaining why social welfare is, as shown in Figure 4, higher in Scenario 2.

The literature on revenue-neutral carbon taxes suggests that externality tax revenues are recycled to lower corporate and/or income taxes to yield double-dividends: better environment quality and less pre-existing distortion due to the reduction in corporate and/or income taxes (Parry and Bento, 2000; Manresa and Sancho, 2005; Chiroleu-Assouline and Fodha, 2006; Bento and Jacobsen, 2007; Dissou and Sun, 2013; Williams III et al., 2014). Here, $\tau_2^* < \tau_1^*$ not necessarily because of more tax revenues generated from externality taxes. Given η , it is possible that a higher externality tax reduces labor supply and thus labor income, shrinking the income tax revenue. Meanwhile, a higher externality tax incents firms' abatement, reducing externality and the associated tax revenue. Therefore, $\tau_2^* < \tau_1^*$ is not necessarily attributed to increased tax revenues. Instead, $\tau_2^* < \tau_1^*$ may simply reflect that an externality tax, not an income tax, is a more efficient mean to combat externality to increase social welfare.

In these two scenarios, the Gini coefficients of income and wealth are close. While Propositions 3 and 4 indicate that the two taxes increase the Gini coefficient of income, it is expected that the Gini coefficients of income and wealth are higher in the two scenarios relative to the baseline model. This numerical exercise indicates that the impact of both policy tools on income inequality are close in magnitude at optimum.

Nevertheless, the impact of the optimal taxation on consumption inequality is qualitatively different. Compared to the baseline model, a higher income tax slightly improves consumption inequality in Scenario 1; in contrast, a higher externality tax significantly deteriorates consumption inequality in Scenario 2, in line with Proposition 3. This discrepancy

in consumption inequality between the two scenarios becomes wider as η increases because τ_1^* and ν_2^* rise with η . These findings highlight two important implications. First, it is more efficient to adopt an externality tax, rather than an income tax, to combat a production externality; nevertheless, an externality (income) tax increases (decreases) consumption inequality, which should also be considered in the prescriptions of the externality tax code. Second, the impacts of the policy constraints become more significant as the public grows more concerned with externality.

In sum, if both taxes are freely adjusted, an externality tax increases with η and an income tax rate hardly reacts to η . Compared to the equilibrium at $\eta = 0$, aggregate consumption, income, and wealth slightly decrease with η at the optimum. Meanwhile, production externality plunges substantially, which is the major channel to increase social welfare. While an increasing concern over externality decreases those inequalities, the response of the corresponding optimal externality tax slightly increases inequalities. In the end, our numerical exercise uncovers that the Gini coefficients of consumption, income, and wealth are rather stable at the optimum for all η in Scenario 2. Therefore, the designation of income and externality tax rates under no policy constraint not only improves social welfare via the reduction in a production externality but also brings about no welfare loss associated with consumption, income, and wealth inequalities.

6.3 Case Scenario 3: $\{\nu, \tau\} \subseteq \{\mathbb{R}_+, \tau_1^*\}$

In this case scenario, an externality tax is freely adjusted, but there exists a policy constraint in refining the pre-existing optimal income tax set in Scenario 1. Given τ_1^* , the optimal externality tax continues to increase with η . Since the income tax rates are identical in Scenarios 1 and 3, the only difference is the higher externality tax in Scenario 3. As expected from Proposition 3, this higher externality tax decreases aggregate consumption, income, and wealth (as shown in Figure 6) but increases the Gini coefficients of consumption, income, and welfare (as shown in Figure 7). When $\eta = 0.1$, consumption, income, and wealth inequalities in Scenario 3, measured in terms of the Gini Coefficient, are 7.94, 6.83, and 3.98 percent higher than the respective inequality in Scenario 1.

A higher externality tax improves social welfare because it incents firms to abate externality. Meanwhile, this higher tax reduces social welfare via at least two channels. It lowers

aggregate consumption, income, and wealth in a stationary equilibrium and makes an economy unequal in terms of the Gini coefficients of consumption, income, and wealth. These channels hurt an economy in terms of social welfare. Since the welfare gains from the reduction in production externalities outweigh the welfare loss due the reduction in aggregate consumption and its inequality, it is beneficial to raise the externality tax in response to the increasing concern over externality.

In reality, income and externality tax rates are not jointly chosen by policymakers. As noted by [Goulder \(1995\)](#), [Bovenberg and Goulder \(1996\)](#) and [Goulder et al. \(1997\)](#), externality taxes are often introduced in the presence of distortionary taxes, including income taxes. If the income tax rates were determined to accommodate concerns over production externality in the first place, the introduction of an externality tax does improve social welfare as measured by an utilitarian welfare criterion. Nevertheless, the imposition of an externality tax in this situation significantly increases the inequalities of consumption, income, and wealth. If households are concerned with aggregate consumption and consumption inequality as in [Dupor and Liu \(2003\)](#), [Abel \(2005\)](#), [Alonso-Carrera et al. \(2008\)](#), [Wendner and Goulder \(2008\)](#), [Wendner \(2010\)](#), and [Bishnu \(2013\)](#), this policy constraint will severely hurt an economy, and the effects on these inequalities will be more pronounced as the concern over production externalities rises. This inequality concern, though important in a political consideration, have not been highlighted in this literature. Moreover, our finding also bridges externality preference to the relationship between optimal taxation and consumption, income, and wealth inequalities.

6.4 Sensitivity Analysis: Risk Aversion

We perform the same numerical simulation to assess the sensitivity of our results to preference parameters. We allow for different degrees of risk aversion for households and report the results of the optimal taxation and Gini coefficients in [Tables 2 and 3](#), respectively.

Two results emerge from [Table 2](#). First, the optimal income tax rate is higher as households become more risk averse regardless of policy constraint and externality preference. Sharing idiosyncratic risk between households would be more valuable if households are more risk averse. Hence, the optimal income tax rate increases with the degree of risk aversion for households, in line with that in [Low and Maldoom \(2004\)](#) and [Hsu and Yang](#)

Table 2: Optimal Taxation: Sensitivity to Risk Preferences

Scenario	Optimal Income Tax	Optimal Externality Tax	Optimal Income Tax	Optimal Externality Tax	Optimal Income Tax	Optimal Externality Tax
	$\eta = 0.05$		$\eta = 0.10$		$\eta = 0.15$	
	Panel A: $\sigma = 1.5$					
1	0.124	0.000	0.168	0.000	0.214	0.000
2	0.097	0.063	0.096	0.109	0.097	0.150
3	0.124	0.060	0.168	0.100	0.214	0.130
	Panel B: $\sigma = 2$					
1	0.140	0.000	0.185	0.000	0.231	0.000
2	0.114	0.064	0.113	0.110	0.115	0.152
3	0.140	0.060	0.185	0.100	0.231	0.130
	Panel C: $\sigma = 2.5$					
1	0.155	0.000	0.199	0.000	0.248	0.000
2	0.129	0.065	0.129	0.111	0.130	0.153
3	0.155	0.060	0.199	0.100	0.248	0.130

(2013). Second, the optimal externality tax rate hardly reacts to a risk preference. Table 2 indicates that the optimal externality tax rates range 0.060-0.065, 0.109-0.111, and 0.130-0.153 when externality preference parameters are 0.05, 0.10, and 0.15, respectively. This finding suggests that the optimal externality tax rate hardly reacts to a risk preference regardless of externality preference.

Several important implications of this paper preserve as a risk preference varies. First, the optimal income tax rate increases as the public grows more concerned with externality in the absence of externality tax (Scenario 1). When η increases from 0.05 to 0.15, τ_1^* increases from 0.124 to 0.214, from 0.140 to 0.231, and from 0.155 to 0.248 at $\sigma = 1.5, 2,$ and 2.5, respectively. In the absence of an externality tax, the income tax τ_1^* should increase to combat an externality as the public is more concerned with externality.

Second, the optimal income tax rate hardly reacts to externality preference in the absence of policy constraint (Scenario 2). τ_2^* ranges 0.096-0.097 (at $\sigma = 1.5$), 0.113-0.114 (at $\sigma = 2$), and 0.129-0.130 (at $\sigma = 2.5$) when η is 0.05, 0.10, and 0.15, respectively. In response to the increasing concern in the absence of policy constraint, the ν_2^* should increase and the τ_2^* should remain unaffected because it is less efficient to combat an externality with

Table 3: Gini Coefficients: Sensitivity to Risk Preferences

The Gini Coefficients of Consumption (C), Income (I), and Wealth (W)									
	C	I	W	C	I	W	C	I	W
	$\eta = 0.05$			$\eta = 0.10$			$\eta = 0.15$		
	Panel A: $\sigma = 1.5$								
1	0.214	0.378	0.540	0.200	0.374	0.536	0.186	0.370	0.531
2	0.223	0.387	0.550	0.222	0.386	0.550	0.222	0.386	0.551
3	0.220	0.391	0.552	0.216	0.399	0.556	0.211	0.407	0.558
	Panel B: $\sigma = 2$								
1	0.213	0.373	0.525	0.199	0.369	0.520	0.185	0.366	0.515
2	0.222	0.382	0.535	0.221	0.382	0.535	0.221	0.382	0.535
3	0.219	0.387	0.538	0.214	0.394	0.541	0.209	0.404	0.544
	Panel C: $\sigma = 2.5$								
1	0.212	0.371	0.512	0.198	0.367	0.507	0.183	0.363	0.502
2	0.220	0.379	0.523	0.220	0.379	0.522	0.219	0.378	0.522
3	0.218	0.384	0.526	0.213	0.391	0.529	0.208	0.401	0.533

an increased income tax. Third, the optimal externality tax rate in Scenario 3 ν_3^* is close to but slightly less than ν_2^* (i.e., the optimal externality tax rate in the absence of policy constraint (Scenario 2)). Interestingly, if an externality tax rate is allowed to be adjusted at the optimal income tax as high as τ_1^* , it remains beneficial to the society by setting the externality tax rate ν_3^* as high as ν_2^* .

Next in line are the implications on consumption, income, and wealth distributions in Table 3. Two important results of this paper preserve as a risk preference varies. First, as households become more risk averse, the Gini coefficients remain consistent with the empirical order as in [Aiyagari \(1994\)](#) and [Hsu and Yang \(2013\)](#): there is less dispersion in consumption and greater dispersion in wealth compared with income. Second, a rise in an externality tax rate increases the Gini coefficients of consumption, income, and wealth. It is noted that the optimal income tax rates are identical in Scenarios 1 and 3, but the optimal externality tax rate is higher in Scenario 3. Meanwhile, Table 3 indicates that those Gini coefficients are higher in Scenario 3 than Scenario 1. This result, as expected from Proposition 3, holds regardless of risk and externality preferences.

7 Conclusion

This paper answers how households and firms behave as the households grow more concerned with negative production externalities, how externality and income taxes should respond to this increasing concern, and how the two taxes affect the distributions of the consumption, income, and wealth. It extends earlier analytical works in a general equilibrium setting by considering production externality in utility and household heterogeneities in productivity, both of which are directly relevant to the problem at hand. Such an extension allows us to uncover the mechanisms through which an increasing concern over production externality and the two taxes affect aggregate consumption, labor supply, externality, and, equally importantly, consumption and income inequalities in a stylized analytical model.

Moreover, this paper incorporates production externality into a utility function and quantitatively characterizes the optimal income and externality taxes under various policy constraints via the framework of [Aiyagari \(1994\)](#), in which households face idiosyncratic earnings risk that cannot be fully insured. Our numerical model measures the empirical significance of these mechanisms in a more realistic model framework.

Six substantive findings emerge from this paper. First, externality could increase as the public grows more concerned with externality. Second, an increasing concern could make the distributions of consumption, income, and wealth more equal. Third, an externality tax decreases aggregate consumption, aggregate output, and externality and increases the Gini coefficients of consumption and income. Fourth, the optimal income tax increases with the concern in the absence of externality taxes (Scenario 1). Fifth, if the two taxes are adjusted under no policy constraints, the optimal income tax hardly reacts to changes in externality preferences, and the externality tax rises with the concern. The policy tools do not change the inequalities of consumption, income, and wealth much at the optimum. Lastly, this paper explores the situation in which externality taxes are adjusted at the optimal income tax rate chosen in Scenario 1. Interestingly, the optimal externality tax sharply increases with concerns over production externality even though the chosen income tax rate rises with the concern. Under this policy constraint, the Gini coefficients of capital and labor incomes skyrocket. In the end, consumption, income, and wealth inequalities become more severe at the optimum. Our numerical analysis indicates that neglecting the policy constraints could lead to the prescription of tax codes that deviate substantially from the optimum and

to the distributional consequences that could be qualitatively and quantitatively different. Moreover, the impacts of policy constraints become more significant if externality arouses more public concern.

8 Appendix: Proof

8.1 The Derivation of Equation (8)

Substituting equation (6) in equation (7), we have

$$L = \phi \tilde{l} - \frac{1 - \phi}{1 - \tau} \left(\tau L + \frac{(\nu - \eta)E}{w} \right).$$

Rearranging terms, we have

$$[(1 - \tau) - (1 - \phi)\tau] L = \phi(1 - \tau)\tilde{l} - (1 - \phi)\frac{(\nu - \eta)E}{w}.$$

Using the first-order conditions (4), we have

$$\frac{E}{w} = \frac{1 - \gamma}{\gamma\nu} L.$$

The above two equations give

$$(1 - \phi\tau)L = \phi(1 - \tau)\tilde{l} - (1 - \phi)\frac{(1 - \gamma)(\nu - \eta)}{\gamma\nu} L.$$

Rearranging terms in the above equation gives equation (8).

8.2 Proof of Proposition 2

Totally differentiating equation (8), it is straightforward to show that $dL^*/d\eta > 0$. By equations (6), (9), and (10), we have $dE^*/d\eta > 0$, $dT^*/d\eta > 0$, and $dw/d\eta = 0$. Hence, $dC^*/d\eta > 0$ from equation (11). Since $dL^*/d\eta > 0$, it is straightforward to see that $dL^*/d\eta = -dB/d\eta > 0$ using equation (7). Hence, $dn^*(l)/d\eta = -l^{-1}dB/d\eta = l^{-1}dL^*/d\eta > 0$ and $dc^*(l)/d\eta > 0$ from equations (1) and (2). Furthermore, $d^2n^*(l)/dl d\eta = -l^{-2}dL^*/d\eta < 0$ because L^* is independent of l .

Totally differentiating the Gini coefficient of consumption (12) with respect to η , we have

$$\frac{dG(c)}{d\eta} = -\frac{G(c)}{C} \frac{dC}{d\eta} < 0.$$

Totally differentiating the Gini coefficient of income (13) with respect to η , we have

$$\frac{dG(z)}{d\eta} = -\frac{G(z)}{L} \frac{dL}{d\eta} < 0.$$

8.3 Proof of Proposition 3

Using the first-order conditions (4), it is straightforward to show that $w^* = \gamma\nu E^*/(1-\gamma)L^*$ and thus $\xi_{E,\nu} - \xi_{w,\nu} = \xi_{L,\nu} - \nu^{-1}$, where $\xi_{a,b}$ is the elasticity of a with respect to b .

Totally differentiating aggregate consumption (11) with respect to ν , we have

$$\begin{aligned} \frac{dC}{d\nu} &= \phi \left[(1-\tau)\tilde{l} + \tau L \right] \frac{dw}{d\nu} + \phi\tau w \frac{dL}{d\nu} + \phi E + \left[\phi\nu + (1-\phi)\eta \right] \frac{dE}{d\nu} \\ \nu \frac{dC}{d\nu} &= \phi \left[(1-\tau)\tilde{l} + \tau L \right] w\xi_{w,\nu} + \phi\tau\nu w \frac{dL}{d\nu} + \phi\nu E + \left[\phi\nu + (1-\phi)\eta \right] E\xi_{E,\nu} \\ \nu \frac{dC}{d\nu} &= \left\{ C - \left[\phi\nu + (1-\phi)\nu \right] E \right\} \xi_{w,\nu} + \phi\tau\nu w \frac{dL}{d\nu} + \phi\nu E + \left[\phi\nu + (1-\phi)\eta \right] E\xi_{E,\nu} \\ \nu \frac{dC}{d\nu} &= C\xi_{w,\nu} + \phi\tau\nu w \frac{dL}{d\nu} + \phi\nu E + \left[\phi\nu + (1-\phi)\eta \right] E(\xi_{E,\nu} - \xi_{w,\nu}) \\ \xi_{C,\nu} - \xi_{w,\nu} &= \frac{\phi\tau\nu w}{C} \frac{dL}{d\nu} + \frac{\phi\nu E}{C} + \left[\phi\nu + (1-\phi)\eta \right] \frac{E}{C} (\xi_{E,\nu} - \xi_{w,\nu}) \\ \xi_{w,\nu} - \xi_{C,\nu} &= -\frac{\phi\tau\nu w}{C} \frac{dL}{d\nu} + \frac{\phi\nu E}{C} \left(-\xi_{L,\nu} + \frac{1-\nu}{\nu} \right) + \frac{(1-\phi)\eta E}{C} \left(-\xi_{L,\nu} + \frac{1}{\nu} \right) > 0 \end{aligned}$$

The last step makes use of the result $\xi_{E,\nu} - \xi_{w,\nu} = \xi_{L,\nu} - \nu^{-1}$. The assumption $\nu \in (0, 1)$ is a sufficient condition to show the last step.

Totally differentiating the Gini coefficient of consumption (12) with respect to ν , we have

$$\frac{dG(c)}{d\nu} = \frac{G(c)}{\nu} (\xi_{w,\nu} - \xi_{C,\nu}) > 0.$$

Totally differentiating the Gini coefficient of labor income (13) with respect to ν , we

have

$$\frac{dG(z)}{d\nu} = -\frac{G(z)}{L} \frac{dL}{d\nu} > 0.$$

8.4 Proof of Proposition 4

Totally differentiating equation (8), it is straightforward to show that $dL^*/d\tau < 0$. By equations (9) and (10), we have $dE^*/d\tau < 0$ and $dw^*/d\tau = 0$.

Totally differentiating aggregate consumption (11) with respect to τ gives us

$$\frac{dC}{d\tau} = \phi \left[(1 - \tau)\tilde{l} + \tau L \right] \frac{dw}{d\tau} + \phi \left[-\tilde{l} + L + \tau \frac{dL}{d\tau} \right] w + \left[\phi\nu + (1 - \phi)\eta \right] \frac{dE}{d\tau}.$$

Since $dw^*/d\tau = 0$, $dL^*/d\tau < 0$, and $dE^*/d\tau < 0$, $L^* \leq \tilde{l}$ implies $dC^*/d\tau < 0$. Note that $L^* = \int n^*(l)ldH(l) \leq \int ldH(l) = \tilde{l}$ because $n^* \in [0, 1]$. Hence, $dC^*/d\tau < 0$.

Totally differentiating the Gini coefficient of labor income (13) with respect to τ , we have

$$\frac{dG(z)}{d\tau} = -\frac{G(z)}{L} \frac{dL}{d\tau} > 0.$$

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