# ENVIRONMENTAL IMPOSSIBILITY THEOREM

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**Abstract**

We investigate the tradeoff between employment and environment quality in the presence of frictional involuntary unemployment. First, we show that it is impossible not to reduce employment if a policy only regulates the polluting sector and does not affect the decision on the intensive margin of emission. Second, we prove that it is impossible for an emission tax or an emission standard policy to increase employment relative to no intervention. However, there always exists an emission cap under which the emission standard policy leads to a higher level of employment and a lower level of emission per output than an emission tax policy for any emission tax rate. Third, we establish conditions under which a revenue-neutral emission tax policy has "double dividend": more employment and better environment quality.

**Keywords:** Frictional Unemployment; Environmental Policies; Double Dividend.

**JEL Classification Numbers:** D62, H23, J64, Q52.

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# 1 Introduction

Environmental policies are often labelled "job-killing" policies. Concerns of adverse employment effect foster political forces that undermine environmental law making and enforcement. However, empirical supports on the adverse employment effect of environmental policies are mixed. While Greenstone (2002), Walker (2011), Curtis (2014), and Yip (2017) find that manufacturing employment declines substantially under various environmental policies, such reduction is found to be small or statistically insignificant in other situations (Berman and Bui, 2001; Martin et al., 2014; Yamazaki, 2017; Hafstead and Williams III, 2016).[1] Despite these political concerns and the extensive debates of the adverse employment effect of environmental policies, we lack a general theoretical framework to understand the employment effects of environmental policies.[2]

This paper therefore purposes to answer two general theoretical questions underlying the employment-environment debate. First, *under what circumstances is the tradeoff between employment and environment quality inevitable?* Second, *can policies increase employment and improve environment quality simultaneously and what conditions are required to achieve these goals?*[3] It is important to answer these questions for two reasons. First, the public and policymakers are mostly concerned about job loss, in addition to welfare loss, in policy debates. While a voluminous literature exists on the welfare analysis of various environmental policies, this paper, in contrast to maximizing specific welfare functions, identifies a broader class

[1] Yamazaki (2017) finds that revenue-neutral carbon taxes increase overall employment in British Columbia, Canada. Hafstead and Williams III (2016) calibrate a general-equilibrium two-sector search model to U.S. economy and find that the net effect on overall employment and unemployment can be small, even in the short run.

[2] Most existing important theoretical works neglect the employment effects (Bovenberg and Goulder, 1996; Parry et al., 1999; Bovenberg and Goulder, 2002; Barrage, 2014; Golosov et al., 2014; Ploeg and Withagen, 2014; Konishi and Tarui, 2015; Goulder et al., 2016). The model of Hafstead and Williams III (2016) relies on the parameter values to generate its quantitative predictions of employment effects and cannot provide a theoretical insight of the problem under general conditions.

[3] Here, "policies" means a broad class of policies that can shift the fundamental parameters of the economy, not limited to commonly seen environmental policies such as a simple emission tax, an emission standard, etc.

of policies that achieve the two objectives: more employment and better environment quality. Second, answering the questions provides a different set of insights on environmental policies as policy tools. It sharpens our understanding of the functioning of various taxes in the labor market and eases the political concerns of the tradeoff between employment and environmental quality. Nevertheless, answering these questions are rather challenging; it requires us to establish general analytical conditions that are independent of particular scenarios or sets of values of parameters as in quantitative analyses in the literature.

This paper incorporates emission decisions on both the extensive and intensive margin of emission into a two-sector search equilibrium model (Acemoglu, 2001; Rogerson et al., 2005; Chassamboulli and Palivos, 2014; Hafstead and Williams III, 2016) to investigate the relationship between employment and emission.[4] In contrast to the existing welfare analyses (Fischer and Springborn, 2011; Holland, 2012; Heutel, 2012; Holland and Yates, 2015; Dissou and Karnizova, 2016; Shinkuma and Sugeta, 2016; Goulder et al., 2016; Hafstead and Williams III, 2016), this paper uses a different approach to select a policy: a policy is preferred to another one if it leads to a higher level of employment and a lower level of emission per output (Employment and Environment preferable, or EE preferable). This selection criterion is independent of the functional form assumptions of the social planner's preference, thus more flexible. Because the increase in employment and reduction in emission will in principle increase the value of a large class of social planner's objective function, our preferable policy will be preferred to a laissez-faire equilibrium under a variety of welfare functions. It should be noted, however, that our criterion does not imply the optimal solution to a social planner's welfare maximization problem. It is possible that a welfare maximizing policy is not EE preferable to a laissez-faire equilibrium because of the weight on employment or emission reduction. Even if the welfare maximizing policy is unique and EE preferable to a laissez-faire equilibrium, it is also possible to have a broader set of policy that is EE preferable. Without specifying a welfare function, it

[4]The extensive margin of emission means the market shares of polluting sector and the intensive margin of emission means the level of emission per filled vacancy in the polluting sector.

is rather difficult to compare between policies selected by our criterion and policies selected by the welfare maximization criterion. Despite this drawback, several interesting findings emerge from our analysis.

First, we prove a theorem that, with job search frictions in the labor market, it is impossible to simultaneously increase employment and reduce emission without adjustment on the intensive margin of emission or intervention in the nonpolluting sector. Regulating the polluting sector increases firms' flow cost and decreases their flow profit, causing their supply of job openings to decline. This reduces the number of vacancies in the polluting sector relative to that in the nonpolluting sector, i.e., the extensive margin of emission. Because the expansion of the nonpolluting sector lowers its price hence profit expectation, firms in this sector will not open sufficient new jobs to offset the loss of jobs in the polluting sector. Consequently, both the employment rate and the extensive margin of emission decline: the tradeoff between employment and environment quality is inevitable. This theoretical relationship between employment and emission is novel and shows the importance of the intensive margin of emission. Nevertheless, the intensive margin of emission and frictional involuntary unemployment are rarely considered in the literature. For example, none of the works in the growing literature that compares between various environmental policies such as emission taxes, emission caps, and intensity targets in DSGE framework consider all the three elements: employment and extensive margin and intensive margin of emission (Fischer and Springborn, 2011; Heutel, 2012; Dissou and Karnizova, 2016).

Second, a number of commonly seen policies are not EE preferable to no intervention. Because there exists no policy that only regulates the polluting sector, does not alter the intensive margin of emission, and is EE preferable to no intervention, it is impossible for corporate taxes, wage taxes, capital taxes, and/or lump-sum taxes in the polluting sector to be EE preferable to a laissez-faire equilibrium. Furthermore, this paper shows that despite reducing the intensive margin of emission, none of the emission standards or simple linear emission taxes are EE preferable to the laissez-faire economy because both of them reduce employment. However, if policymakers have to select a policy between them, our theory indicates that for any positive emission tax rate there always exists an interval of the emission standards such that the emission standards are EE preferable to linear emission taxes.

Third, this paper demonstrates that a revenue-neutral emission tax could be EE preferable to no intervention. In other words, under certain intuitive conditions, there exists an emission tax rate under which a revenue-neutral emission taxation scheme reduces pollutant emission and simultaneously increases employment. On the one hand, the emission taxes shrink the polluting sector and the nonpolluting sector through the general equilibrium effect on the relative price. On the other hand, the transfer of the tax revenue increases profits and expands the nonpolluting sector. If the price elasticity of the clean goods with respect to the tax revenue is sufficiently low, the expansion effect dominates and employment increases in response to the revenue-neutral emission tax. In addition, we show that a revenue-neutral emission tax is shown to be EE preferable to linear emission taxes for any positive tax rate but is only EE preferable to emission standards over certain intervals of the standards. Hence, this paper provides an analytical support for the "double dividend" hypothesis: a revenue-neutral emission taxation scheme increases employment and reduces emission.[5]

Our work is closely related to Wagner (2005) and Hafstead and Williams III (2016), but differs with both papers in a number of ways. First, in addition to the purpose in Wagner (2005), which is to pin down the condition under which employment increases under a more stringent environmental policy, our paper aims to identify the circumstances under which the tradeoff between employment and the environment is inevitable. Our paper is also different from Wagner (2005) in the model assumption. First, Wagner (2005) shuts down the demand response in the goods market in response to environmental policies. However, this assumption is restrictive: Rivers and Schaufele (2015) provide empirical support on a large decline in gasoline demand under the British Columbia's revenue-neutral carbon taxes. Furthermore, we show that

[5] In the literature, the "double dividend" of revenue-neutral emission taxation schemes refers to better environment quality and more consumption. In this paper, employment is viewed as the measure of welfare.

the demand response is one of the key general equilibrium effects leading to an ambiguous employment effect under revenue-neutral emission taxes. Second, Wagner (2005) restricts the means of emission abatement to an independent abatement sector and shows that employment cannot increase under a low emission tax because the abatement sector is not big enough to employ all the workers that flow out of the regulated sector. On the contrary, our model does not restrict the means of abatement. We allow the polluting sector to abate the emission level by switching to a cleaner fuel source or employing emissions control technology. Our paper shows that even if emission abatement creates no new job, there still exists a positive emission tax rate such that a revenue-neutral emission taxation scheme could increase employment.

Second, unlike Hafstead and Williams III (2016), our paper provides theoretical predictions of the employment effect of a broad class of environmental policies under general conditions. The model of Hafstead and Williams III (2016) and this paper share identical essential ingredients, namely, labor market search friction and emission abatement decisions, and our theoretical predictions about the policies complement their paper. The advantage of our paper is that our results do not depend on specific calibrations and the results apply to a larger set of policies and situations than those considered in Hafstead and Williams III (2016). In addition, our results hold without adjustment on the margin of labor supply, i.e., under exogenous labor supply, suggesting that labor supply decisions of households, a feature of the model of Hafstead and Williams III (2016), is not the key factor in considering the tradeoff between employment and environment quality.

This paper proceeds as follows. Section 2 introduces the model. Section 3 describes the steady state equilibrium, defines the criteria for which a policy is EE preferable, and establishes the relationship between employment and the extensive margin of emission. Section 4 compares amongst emission taxes, emission standards, and no intervention. Section 5 compares a revenue-neutral emission tax with emission taxes, emission standards, and no intervention. Section 6 concludes the paper.

## 2 The Basic Model

Our theory is presented through the workhorse of a search equilibrium model, where the frictional unemployment and the extensive and the intensive margin of emission are endogenized. We aim to construct a simple model to shed light on the mechanism through which policies influence the frictional unemployment and the extensive and the intensive margin of emission. Despite the abstraction of other non-essential goods and labor market features, our model is flexible and could be easily extended to incorporate those other features.

Our model is formulated in continuous time and assumes the steady state of the labor market environment. Consider an economy with a continuum of utility maximizing workers and profit maximizing vacancies. Both workers and vacancies are risk neutral and discount the future with an identical real interest rate $r$.[6] Workers are identical and are infinitely lived, but unfilled vacancies can be destroyed according to its profit condition. Without loss of generality, the mass of workers is normalized to unity and the mass of vacancies is endogenized. All workers derive their utility from the final consumption good $Y$ and maximize the present discounted value of their utility.

**Good Market.** We assume the final consumption good is a homogeneous composite good that aggregates sectoral heterogeneity.[7] This allows us to interpret it as a utility index when appropriate.

Production of the final good requires input of two intermediate goods with technology given

[6]This simplification is common in this literature (Mortensen and Pissarides, 1994; Moen, 1997; Moscarini, 2005; Rogerson et al., 2005; Gonzalez and Shi, 2010; Fujita and Ramey, 2012; Michaillat, 2012). Applications of the search and matching model also assume agents to be risk-averse, such as the literature that investigates the optimal unemployment benefits with search frictions (Fredriksson and Holmlund, 2006; Guerrieri et al., 2010). Recent literature also investigates job search behaviors with the preference of ambiguity aversion (Chan and Yip, 2017). Our model can be easily extended to incorporate agents' ambiguity aversion; we do not do so because the modification of agents' preference towards ambiguity complicates our model without providing a richer economic intuition in our context.

[7]This simplification is adopted in search and matching literature (Acemoglu, 2001; Chassamboulli and Palivos, 2014). A similar constant elasticity of substitution (CES) aggregation function is also used in Hafstead and Williams III (2016).

by

$$Y = \left(\alpha Y_d^{\rho} + (1-\alpha) Y_g^{\rho}\right)^{\frac{1}{\rho}} \tag{1}$$

where $Y_j$ is the output of intermediate good $j$, $j = d$ ($j = g$) indexes output of the polluting "dirty" sector (the nonpolluting "green" sector). We will call $Y_d$ the dirty good and $Y_g$ the clean good for simplicity. The elasticity of technical substitution between $Y_d$ and $Y_g$ is constant and equal to $1/(1-\rho)$, with $\rho < 1$. $\alpha$ parameterizes the relative importance of $Y_d$ in production of $Y$.

We assume the markets for the intermediate goods are perfectly competitive and normalize the price of $Y$ to unity. Hence, the prices of the intermediate goods are

$$p_d = \alpha \left(\frac{Y}{Y_d}\right)^{1-\rho}, \; p_g = (1-\alpha)\left(\frac{Y}{Y_g}\right)^{1-\rho} \tag{2}$$

Hence, $p_j$ declines when the supply of the corresponding intermediate goods $Y_j$ increases.

**Labour Market.** Workers are either employed or unemployed. Each worker can at most work in one vacancy, and each vacancy can at most be filled by one worker.[8]

Vacancies are either filled or unfilled and are created in either the polluting sector or the nonpolluting sector. To create a vacancy, sector-specific machinery is required so that vacancies, once created, are not allowed to switch between sectors. Machinery costs $k$ regardless of sector. Since a vacancy can be filled by at most one worker, the machinery cost can be viewed as an sector-specific capital-labour ratio or capital intensity. Vacancies in the two sectors produce two different nonstorable intermediate goods.

[8]The model assumes that no decision on labor supply, either the number of working hours or labor force participation, is made. This simplification is standard (Mortensen and Pissarides, 1994; Shimer, 2005; Hall, 2005; Hagedorn and Manovskii, 2008; Hall and Milgrom, 2008; Fujita and Ramey, 2012; Michaillat, 2012), and is in line with empirical regularities: cyclical variations in total working hours (unemployment) basically arise from changes in the number of employment but not changes in working hours per worker (labor force participation) (Shimer, 2010).

Unemployed workers and unfilled vacancies come together via a matching technology $M(u,v)$, where $u$ is the unemployment rate and $v$ is the measure of unfilled vacancies. Following the literature, we assume $M(u,v)$ is twice differentiable and increasing in both its arguments, and exhibits constant returns to scale. In each instant, the flow rate of match for an unfilled vacancy is $q(\theta) \equiv M(u,v)/v$, where $q(\theta)$ is a differentiable decreasing function, and $\theta \equiv v/u$ is market tightness. It follows immediately that $M(u,v)/u = \theta q(\theta)$ is the flow rate of match for an unemployed worker. It can be shown that $\theta q(\theta)$ is increasing in $\theta$. The standard Inada-type assumptions on $M(u,v)$ is made so that $\lim_{\theta \to 0} q(\theta) = \infty$, $\lim_{\theta \to \infty} q(\theta) = 0$, $\lim_{\theta \to 0} \theta q(\theta) = 0$, and $\lim_{\theta \to \infty} \theta q(\theta) = \infty$.

Upon matching, the employed worker and the filled vacancy bargain on the wage rate $w_j$. After bargaining, production starts in the next instant. Each vacancy in sector $j$ produces $A$ units of $Y_j$, sells the product for price $p_j$, and pays the worker the wage rate $w_j$. The match pair continues to produce until a separation shock arrives.

**Bellman Equations.** Let $J_j^E$ be a worker's value of employment in sector $j$ and $J^U$ his value of unemployment. In sector $j$, an employed worker receives a wage $w_j$. Each match separates at a rate $\lambda$. When the separation shock arrives, the employed worker becomes unemployed. Then, $J_j^E$ can be written as

$$rJ_j^E = w_j + \lambda(J^U - J_j^E) \tag{3}$$

An unemployed worker receives unemployment benefits $z$ and matches with an unfilled vacancy at the rate $\theta q(\theta)$.[9]

$$rJ^U = z + \theta q(\theta) \left[ \phi(J_d^E - J^U) + (1-\phi)(J_g^E - J^U) \right] \tag{4}$$

where $\phi$ is the proportion of unfilled vacancies that are in sector $d$ among all vacancies.

[9]Endogenizing search intensity in this model is simple but would not result in a richer economic intuition. No results in this paper would be altered by endogenizing search intensity.

Let $J_j^F$ and $J_j^V$ be respectively the asset value of a filled and an unfilled vacancy in sector $j$. A filled vacancy receives revenue $p_j A$, pays a wage $w_j(1+\tau_w)$, and separates with the paired worker at a rate of $\lambda \in (0,1)$, where $A$ is the productivity, and $\tau_w \in [0,1)$ is the wage tax rate.[10] The filled vacancy in sector $d$ also pays an emission tax $T_x(x)$ and an abatement cost $c(\bar{x}-x)$, where $x$ is the emission level, $T_x(x)$ is the total tax on emission, and $\bar{x}-x$ is the level of abatement.[11] Denote $\tau_x(x)$ the effective emission tax rate, $\tau_x(x) = T_x(x)/x$; that is, $\tau_x(x)$ is the average emission tax for emission level $x$. In a linear tax scheme, this is also the marginal tax rate on emission; when the tax scheme is nonlinear, $\tau_x(x)$ is the average of marginal tax rates in different ranges of emission level. Thus our presentation accommodates all emission tax schemes.[12] The filled vacancy in sector $d$ also receives an abatement subsidy $S(\bar{x}-x)$. It chooses its abatement level $\bar{x}-x$ to maximize this value. Hence, the asset value of a filled vacancy in sector $j$ is given by

$$\begin{aligned} rJ_d^F &= \max_x \left\{ p_d A - w_d(1+\tau_w) - c(\bar{x}-x) - \tau_x(x)x + S(\bar{x}-x) + \lambda(J_d^V - J_d^F) \right\} \\ rJ_g^F &= p_g A - w_g(1+\tau_w) + \lambda(J_g^V - J_g^F) \end{aligned} \tag{5}$$

We make assumptions on the abatement cost function, the emission tax rate function, and the subsidy function. First, the abatement cost function $c : [0,\bar{x}] \longmapsto \mathbb{R}_+$ is twice differentiable. The cost is equal to zero in the absence of abatement; hence, $c(0) = 0$. Also, the abatement cost and its marginal cost increase with the level of abatement. That is, $c'(\bar{x}-x) > 0$ and

[10] The model assumes that workers are hired only for production. Readers who are interested in theoretical frameworks in which workers engage in recruitment are referred to Hafstead and Williams III (2016).

[11] The choice of abatement method is not explicitly modelled because it will complicate the model without providing richer economic intuitions. Hence, our model does not restrict the means of abatement. In practice, the emission level could be abated by switching to a cleaner fuel source or employing emissions control technology. Interested readers are referred to Wagner (2005).

[12] Our model does not include a consumption tax, but could be easily extended to consider the consumption tax effect. The welfare analysis on the emission tax/emission standard and consumption tax combination is examined in Fullerton (1997) and Holland (2012).

$c''(\bar{x}-x) > 0$. The standard Inada-type assumption is also made so that $\lim_{x\to\bar{x}} c'(\bar{x}-x) = 0$ and $\lim_{x\to 0} c'(\bar{x}-x) = \infty$.

Second, we assume that $\tau_x'(x) \geq 0$ and $\tau_x''(x) \geq 0$ in light of the convexity/progressivity of the popular emission tax scheme. If $\tau_x'(x) = 0$, the tax rate is linear.[13] Third, the abatement subsidy function $S : [0,\bar{x}] \longmapsto \mathbb{R}_+$ is twice differentiable. A vacancy cannot claim any subsidy if the abatement level is zero. That is, $S(0) = 0$. Also, we assume $S'(\bar{x}-x) \geq 0$ to capture the potential progressivity in abatement subsidy. Assume $S'(\bar{x}-x) < c'(\bar{x}-x)$ and $S''(\bar{x}-x) < c''(\bar{x}-x)$ for all $x \in [0,\bar{x}]$ to ensure that the abatement subsidy will be claimed up to the abatement cost. That is, the net cost of abatement is nonnegative.

To maximize the value function, a vacancy in sector $d$ chooses the optimal emission level $x^*$ that solves the following first order equation:

$$\underbrace{c'(\bar{x}-x^*) - S'(\bar{x}-x^*)}_{\text{Marginal Abatement Cost}} = \underbrace{\tau_x'(x^*)x^* + \tau_x(x^*)}_{\text{Marginal Abatement Benefit}} \tag{6}$$

where $x^*$ is the optimal emission level per vacancy. Intuitively, an additional unit of abatement increases abatement cost, but the cost is partially subsidized by the government. So, $c'(\bar{x}-x^*) - S'(\bar{x}-x^*)$ is the net marginal cost of abatement paid by a vacancy. On the right side, a marginal increase in abatement at emission level $x^*$ reduces the total tax payment up to $x^*$ by $\tau_x'(x^*)x^*$ plus saves $\tau_x(x^*)$, which is the tax payment on the marginal emission unit at $x^*$. The optimal $x^*$ thus equates marginal cost of abatement to its marginal benefit.

An unfilled vacancy matches an unemployed worker at rate $q(\theta)$. Hence, we have

$$rJ_j^V = q(\theta)\left(J_j^F - J_j^V\right) \tag{7}$$

[13] $\tau_x(x)$ sums the information on differential marginal tax rates. Suppose a taxation scheme is $\tau_1$ for $x \in [0, x_0]$ and $\tau_2$ for additional units of emission if $x > x_0$. For emission level $x \in [0, x_0]$, $T(x) = \tau_1 x$ and $\tau_x(x) = \tau_1$; for $x > x_0$, $T(x) = \tau_1 x_0 + \tau_2(x - x_0)$, $\tau_x(x) = \tau_2 + \frac{x_0(\tau_1-\tau_2)}{x}$. $\tau_x'(x) = -\frac{x_0(\tau_1-\tau_2)}{x^2}$ if $0 \leq \tau_1 < \tau_2$ (progressive scheme). $\tau_x'(x) = 0$ if $\tau_1 = \tau_2$ (linear scheme). Thus, we only need the sign of $\tau_x'$ to determine tax progressivity.

When a worker and a vacancy meet, they bargain on the wage to maximize the generalized Nash product as follows:

$$w_j \equiv \arg\max \left(J_j^E - J^U\right)^{\beta} \left(J_j^F - J_j^V\right)^{1-\beta}$$

where $\beta \in (0,1)$ is the worker's bargaining power. Using equations (3) and (5), the wage is a solution of the following sharing rule:

$$(1+\tau_w)(1-\beta)(J_j^E - J^U) = \beta(J_j^F - J_j^V) \tag{8}$$

Intuitively, a worker's matching surplus, $J_j^E - J^U$, is a fraction of the total matching surplus. An assumption of free entry and exit condition ensures that the asset value of an unfilled vacancy equals its opening cost. Thus,

$$J_j^V = k \tag{9}$$

where $k$ is the machinery cost.

In a steady state, the flow out of employment equals the flow into employment. Hence, the steady state employment $e$ and unemployment $u$ are given by

$$e = \frac{\theta q(\theta)}{\lambda + \theta q(\theta)},\ u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{10}$$

# 3 Characterization of the Steady-State Equilibrium

This section characterizes the steady state equilibrium of our model, defines the measurements of the environment quality and various policy spaces, and provides economic intuitions behind the relation between employment and the extensive margin of emission. In the end of this section, we will present our Environmental Impossibility Theorem and discuss its implication in practice.

## 3.1 Existence of A Unique Steady-State Equilibrium

This section shows the existence of a unique steady-state equilibrium in our economy. We begin by defining the steady state equilibrium.

**Definition 1.** *A steady state equilibrium is defined as* $\{Y, p_j, w_j, x, e, u, \phi, \theta, J_j^E, J^U, J_j^F, J_j^V\}$ *such that for* $j = \{d, g\}$,

1. *(Production of Final Goods):* $Y$ *satisfies equation (1);*
2. *(Goods Market Clearing):* $p_j$ *satisfies equation (2);*
3. *(Value Functions):* $J_j^E$, $J^U$, $J_j^F$, *and* $J_j^V$ *satisfy equations (3), (4), (5), and (7);*
4. *(Optimal Emission Level):* $x$ *satisfies equation (6);*
5. *(Rent Sharing):* $w_j$ *satisfies the sharing rule (8);*
6. *(Free Entry and Exit):* $\phi$ *and* $\theta$ *satisfy equation (9);*
7. *(Steady-State Accounting):* $e$ *and* $u$ *satisfy equation (10).*

Since both types of vacancies separate with workers at the same rate, $Y_d = e\phi A$ and $Y_g = e(1-\phi)A$ in the steady state. Recall that $\phi$ is the proportion of unfilled vacancies that are in sector $d$ among all vacancies. In the steady state, there are $\phi e$ filled vacancies in sector $d$. Hence, $\phi = Y_d/(Y_d + Y_g)$ can also be viewed as the market share of sector $d$. Using equation (2), the steady state price levels are given by

$$\begin{aligned} p_d(\phi) &= \alpha \left\{ \frac{[\alpha\phi^\rho + (1-\alpha)(1-\phi)^\rho]^{\frac{1}{\rho}}}{\phi} \right\}^{1-\rho} \\ p_g(\phi) &= (1-\alpha) \left\{ \frac{[\alpha\phi^\rho + (1-\alpha)(1-\phi)^\rho]^{\frac{1}{\rho}}}{1-\phi} \right\}^{1-\rho} \end{aligned} \tag{11}$$

Unsurprisingly, $p_d$ is strictly decreasing in $\phi$ and $p_g$ is strictly increasing in $\phi$ because the higher is $\phi$, the more abundantly the dirty good is supplied in the steady state relative to the clean good, lowering its price relative to the price of the clean good.

Using equations (3), (5), (8), and (9), the wage equation is given by

$$\begin{aligned}(1+\tau_w)w_d &= (1+\tau_w)(1-\beta)rJ^U + \beta\left(p_d A - c(\bar{x}-x^*) + S(\bar{x}-x^*) - \tau_x(x^*)x^* - rk\right) \\ (1+\tau_w)w_g &= (1+\tau_w)(1-\beta)rJ^U + \beta\left(p_g A - rk\right) \qquad (12)\end{aligned}$$

where $x^*$ is the optimal emission level that solves equation (6). A worker is compensated with an outside option value plus a fraction of the flow profit in the corresponding sector. Using equations (4), (7), (8), and (9), we have

$$rJ^U = z + \theta\frac{\beta}{(1+\tau_w)(1-\beta)}rk. \qquad (13)$$

$rJ^U$ is increasing in both $\theta$ and $k$. Intuitively, higher market tightness shortens unemployment spell, increasing a worker's outside option value. A zero-profits condition ensures that a rise in the capital-labour ratio $k$ increases a vacancies' outside option value, and thus the expected flow profits. Since a worker is rewarded with a fraction of the flow profit and $rJ^U$ is a function of the expected wage, an increase in the capital-labour ratio will improve $rJ^U$ in the steady state equilibrium.

Using equations (7), (8), (12), and (13), the zero-profits conditions are as follows:

$$\begin{aligned}rk &= \frac{q(\theta)(1-\beta)}{r+\lambda}\{p_d(\phi)A - c(\bar{x}-x^*) + S(\bar{x}-x^*) - \tau_x(x^*)x^* - rk \\ &\quad -(1+\tau_w)[z+\theta\frac{\beta}{(1+\tau_w)(1+\beta)}rk]\} \qquad (14)\end{aligned}$$

$$rk = \frac{q(\theta)(1-\beta)}{r+\lambda}\left(p_g(\phi)A - rk - (1+\tau_w)\left(z+\theta\frac{\beta}{(1+\tau_w)(1+\beta)}rk\right)\right) \qquad (15)$$

Along each locus, vacancies in the corresponding sector make zero profits. Using the two zero-profits conditions (14) and (15), the revenue differential between the two sectors is given by

$$\Lambda(\phi^*) \equiv p_d(\phi^*)A - p_g(\phi^*)A = \underbrace{\overbrace{c(\bar{x} - x^*)}^{\text{Abatement Cost}} - \overbrace{S(\bar{x} - x^*)}^{\text{Abatement Subsidy}}}_{\text{Expenditure on Abatement}} + \underbrace{\tau_x(x^*)x^*}_{\text{Emission Tax}} \quad (16)$$

which is positive. Since the flow cost is higher in sector $d$, its revenue has to be higher in the steady state equilibrium to ensure the zero-profits conditions.

The next proposition shows the existence of a unique steady state equilibrium.

**Proposition 1.** *There exists a unique steady state equilibrium, as defined in Definition 1. In the equilibrium,* $p_d(\phi^*) > p_g(\phi^*)$.

*Proof.* See Appendix. □

## 3.2 Environment Measures and Environmental Policies

This section introduces two measures on the quality of the environment, and defines the criteria to select a policy.

In the steady state equilibrium, there are $\phi^* e^*$ filled vacancies in sector $d$, and each of them emits at the level of $x^*$. Hence, the total emission level $\chi$ is given by

$$\chi \equiv x\phi e \quad (17)$$

This is an absolute measure of emission. Given the number of filled vacancies in sector $d$, an increase in the emission level per vacancy $x^*$ results in a higher level of total emission. Analogously, the more the number of filled vacancies ($\phi e$) in sector $d$, the higher is the level of total emission. $x$ is the intensive margin of emission, and $\phi$ the extensive margin of emission.

In fact, a rise in the total emission level can also be a consequence of the increase in output (i.e. employment rate) even if both the extensive margin $\phi$ and the intensive margin $x$

of the emission level remain unchanged. In this case, the rise in the total emission level has both positive and negative effect on welfare. To take out the output effect, we need a measure in relative terms. Thus, we introduce another measure of the quality of the environment as follows:

$$\varphi \equiv \frac{\chi}{Y} \tag{18}$$

which is emission per output. Therefore, $\varphi$ could be a better measure of the quality of the environment that takes the total output level into account. Using equations (1) and (17), we have

$$\varphi = \frac{x\phi e}{(\alpha Y_d^\rho + (1-\alpha)Y_g^\rho)^{\frac{1}{\rho}}} = \frac{x\phi}{A\left(\alpha\phi^\rho + (1-\alpha)(1-\phi)^\rho\right)^{\frac{1}{\rho}}} \tag{19}$$

where $\varphi(x, \phi)$ increases with $x$ and $\phi$ in a steady state equilibrium.

Before we select the preferred environmental policy, we define the concept of *prefer to* and *strictly prefer to* as follows.

**Definition 2.** *A policy $\mathcal{P}$ is Employment and Environment preferable (EE preferable) to another policy $\mathcal{Q}$ iff $e^*_{\mathcal{P}} \geq e^*_{\mathcal{Q}}$ and $\varphi^*_{\mathcal{P}} \leq \varphi^*_{\mathcal{Q}}$, but not both equal. A policy $\mathcal{P}$ is strictly EE preferable to another policy $\mathcal{Q}$ iff $e^*_{\mathcal{P}} \geq e^*_{\mathcal{Q}}$ and $\chi^*_{\mathcal{P}} \leq \chi^*_{\mathcal{Q}}$, but not both equal.*

By Definition 2, a policy is *strictly* EE preferable if the policy leads to a higher level of employment and a lower level of *total emission*. Analogously, a policy is EE preferable if the policy results in a higher level of employment and a lower level of *emission per output*. It is straightforward to show that the relation *strictly prefer to* implies *prefer to*, and is summarized in the following lemma.

**Lemma 1.** *If a policy $\mathcal{P}$ is strictly EE preferable to a policy $\mathcal{Q}$, the policy $\mathcal{P}$ is EE preferable to the policy $\mathcal{Q}$.*

*Proof.* See Appendix. □

Lastly, in order to understand whether a policy is better or worse than no intervention, we define the situation where the government chooses not to intervene an economy as follows.

**Definition 3.** *A null policy $\varnothing$ is defined as the situation where the government does not intervene an economy.*

In fact, a null policy is one of the policies that can be chosen by the government. Under the null policy, $\tau_x(x) = 0 \; \forall x \geq 0$ and $\tau_w = 0$. We denote $\Omega_\varnothing \equiv \{x^*_\varnothing, \phi^*_\varnothing, \theta^*_\varnothing, \chi^*_\varnothing, e^*_\varnothing\}$ as the set of variables of our interest under the policy $\varnothing$. It is straightforward to see from equation (6) that $x^*_\varnothing = \bar{x}$, $c(\bar{x} - x^*_\varnothing) = 0$ and $S(\bar{x} - x^*_\varnothing) = 0$. $\phi^*_\varnothing$ solves the revenue differential equation $\Lambda(\phi) = 0$. $\Lambda'(\phi) < 0$ implies that $\phi^*_\varnothing$ is unique. Given $\phi^*_\varnothing$, $\theta^*_\varnothing$ solves the zero-profits condition (15). Using equation (10), a unique $e^*_\varnothing$ is pinned down with $\theta^*_\varnothing$. Therefore, all elements in $\Omega_\varnothing$ are solved. Hence, an economy with no government intervention is a special case of this model. We will show that not many environmental policies are EE preferred to $\varnothing$.

## 3.3 Tradeoff between Employment and Environment

In this section, we establish the relationship between employment and the emission on its extensive margin in a steady state equilibrium. In fact, the equilibrium can also be represented using the system of zero-profits equations (14) and (15) rather than via the revenue differential (16).

When market tightness increases, it exerts two forces on expected profits. A rise in $\theta$ reduces the transition rate for a vacancy from being unfilled to being filled; therefore, it lowers unfilled vacancies' expected profits. Meanwhile, a larger $\theta$ increases the job finding rate, shortening the unemployment spell and thus improving unemployed workers' outside option. As a result, filled vacancies are to pay higher wages to workers to compensate for a higher $J^U$. This further lowers profits for filled vacancies. The combination of these two forces hence shrinks the expected profits regardless of sector.

An increase in the share of vacancies in sector $d$ decreases (increases) $p_d$ ($p_g$) and thus

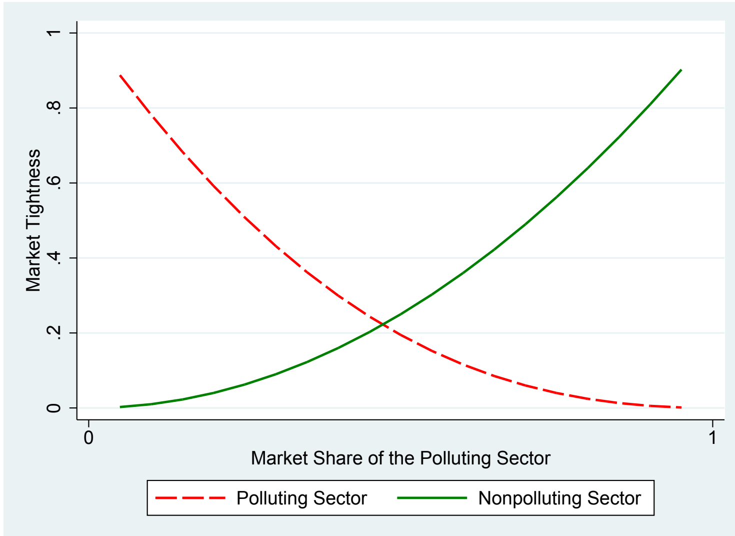


Figure 1: Equilibrium Determination in the $\theta$-$\phi$ Plane

decreases (increases) the revenue in sector $d$ (sector $g$). In response to an increase in $\theta$, $\phi$ has to fall to maintain the zero-profits condition (14) but rise to maintain the condition (15). Hence, the loci (14) and (15) slope downward and upward, respectively, in the $\theta$-$\phi$ plane as demonstrated in Figure 1. When $\phi$ tends to zero, $\theta$ approaches positive infinity in locus (14) but zero in locus (15). On the other hand, when $\phi$ approaches 1, $\theta$ approaches zero in locus (14) and positive infinity in locus (15). By the continuity of the two functions, the two loci intersect only once, where $\phi^* \in (0, 1)$. This is a graphical proof of the first component of Proposition 1.

Employment is strictly increasing in $\theta$ according to equation (10). Moreover, $e$ tends to one (zero) when $\theta$ approaches positive infinity (zero). We can therefore map the equilibrium conditions (14) and (15) from the $\theta$-$\phi$ plane to the $e$-$\phi$ plane as shown in Figure 2, where the loci (14) and (15) slope download and upward, respectively. Any policy that only regulates sector $d$ shifts locus (14), not the locus (15). As a result, the steady state equilibrium moves along locus (15), which positively relates $e$ to $\phi$, and thus $e$ to $\phi e$. This is the primary force that trades employment $e$ off against the extensive margin of the total emission level $\phi e$.

The following theorem summarizes the relationship between employment and the extensive margin of emission.

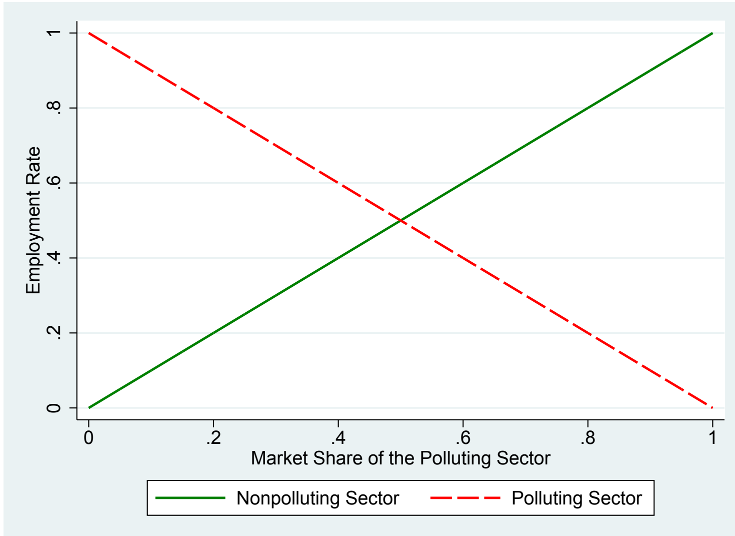


Figure 2: Equilibrium Determination in the $e$-$\phi$ Plane

**Theorem 1.** ***(Employment-Environment Tradeoff Theorem)*** *Denote $\Psi$ as a class of all policies $\mathcal{P}$ that do not regulate the nonpolluting sector and do not alter $x$. Then, for all $\mathcal{P} \in \Psi$, either $e^*_{\mathcal{P}} \geq e^*_{\varnothing}$ and $\phi^*_{\mathcal{P}} \geq \phi^*_{\varnothing}$ or $e^*_{\mathcal{P}} < e^*_{\varnothing}$ and $\phi^*_{\mathcal{P}} < \phi^*_{\varnothing}$.*

*Proof.* See the discussion above. □

Theorem 1 is important (even crucial) in understanding the impacts of any environmental regulation on the environment and employment. Consider a model that shuts down firms' emission decision on its intensive margin. That is, $x > 0$ is exogenously given. According to Theorem 1, any environmental policy that changes the flow cost in the polluting sector will shift the locus (14) along the locus (15). The tradeoff between the environment and the employment is destined in the model regardless of the measure of the quality of the environment. The theorem illustrates that the intensive margin of emission, which is often ignored in the literature, is an essential ingredient for the tradeoff between environment and employment. Thus, it is essential for a theoretical model to endogenize the emission level from both its intensive and extensive margin to fully understand the consequences of a policy. Without adjustment on the intensive margin, all policies that only regulate the polluting sector will trade employment off against the quality of environment. Moreover, such tradeoff also leads to the following theorem.

**Theorem 2.** ***(Environmental Impossibility Theorem)*** *Denote $\Psi$ as a class of policies $\mathcal{P}$, all of which do not regulate the nonpolluting sector, and result in the same emission level on the intensive margin in the polluting sector. Then, $\mathcal{P}$ is not EE preferable to $\mathcal{Q}$, for all $\mathcal{P}, \mathcal{Q} \in \Psi$. In particular, if $\varnothing \in \Psi$, there exists no $\mathcal{P} \in \Psi$ such that $\mathcal{P}$ is EE preferable to $\varnothing$.*

This theorem indicates that if two environmental instruments regulate only the polluting sectors and yield the same emission per vacancy, they are not EE preferable to each other. In other words, these classes of policies either have a lower employment level or a higher measure of emission than one another. In particular, if a class of environmental instruments that yield an identical emission per vacancy as $\varnothing$, they are not EE preferable to $\varnothing$. Notice that yielding the same emission per vacancy as $\varnothing$ simply means the policies do not alter the intensive margin of emission. Therefore, the employment-environment tradeoff theorem and the impossibility theorem imply that only the policies that depress the emission level on its intensive margin could be preferred to $\varnothing$. In practice, the commonly used policy instruments, such as the imposition of higher corporate taxes, wage taxes, capital taxes or lump-sum taxes on the polluting sector, do not regulate the nonpolluting sector or alter the optimal emission level per vacancy; thus, the impossibility theorem predicts that these policies are not EE preferable to $\varnothing$.

However, if a policy alters the intensive margin of emission, it does not necessarily imply that the policy is EE preferable to $\varnothing$. We will show that the two commonly seen environmental policies are not EE preferable to $\varnothing$; neither a simple emission taxation scheme nor an emission standard is EE preferable to $\varnothing$ even though both of them reduce the intensive margin of emission.

# 4 Emission Tax and Emission Standard

Up to this point, we have compared the equilibria under various classes of policies to a laissez-faire equilibrium. In fact, our defined preferences, namely EE preferable, can be widely used to compare various policies to the null policy and to compare between two different classes of policies. In particular, this section compares the impacts of emission taxes and emission standards using the two criteria, employment and environment quality, and compare the variables of interest under the two classes of policies to understand the differences in the mechanism through which the two policies affect the labor market.

In this section, we first show that both the emission taxes and the emission standards, even though reducing the intensive margin of emission, are not EE preferable to a laissez-faire equilibrium. Second, we demonstrate that for all positive emission taxes, there always exists an interval of emission caps under which an emission standard policy is EE preferable to an emission tax policy.

## 4.1 Emission Tax

We examine the effects of the levy of a linear emission tax on the otherwise laissez-faire equilibrium. We denote $\mathcal{ET}(\tau_x)$ as the collection of all linear emission tax policies with $\tau_x \in \mathbb{R}_{++}$. To minimize $c(\bar{x} - x) + \tau_x x$, the optimal emission level solves the following first order condition:

$$c'(\bar{x} - x^*_{ET}) = \tau_x \tag{20}$$

Using similar arguments as in equation (6), there exists a unique $x^*_{ET} \in (0, \bar{x})$ that solves the first order condition (20). As shown before, $x^*_{\varnothing} = \bar{x}$ and thus we can conclude that $x^*_{ET} < x^*_{\varnothing}$. The revenue differential analogous to equation (16) is:

$$\Lambda(\phi^*_{ET}) = \underbrace{c(\bar{x} - x^*_{ET})}_{\text{Abatement Cost}} + \underbrace{\tau_x x^*_{ET}}_{\text{Emission Tax}} \tag{21}$$

The differential stems from the abatement cost and the emission tax. Totally differentiating equation (21), we have

$$\frac{d\phi^*}{d\tau_x} = x^*_{ET} \left(\Lambda'(\phi^*_{ET})\right)^{-1} < 0 \tag{22}$$

A rise in the emission tax increases the flow cost of vacancies in sector $d$, lowering their flow profit, and thus depreciating the value of $J_d^V$. Hence, some of the unfilled vacancies in sector $d$ leave the market until $J_d^V$ equals $k$. Consequently, the fraction of vacancies in the polluting sector falls. Theorem 1 implies that employment declines.

In response to the increase in the emission tax, the change in the total emission level is given by

$$\frac{d\chi^*}{d\tau_x} = \underbrace{\phi^* e^* \frac{dx^*}{d\tau_x}}_{\text{Effect on Intensive Margin}} + \underbrace{\overbrace{x^* e^* \frac{d\phi^*}{d\tau_x}}^{\text{Substitution Effect}} + \overbrace{x^* \phi^* \frac{de^*}{d\tau_x}}^{\text{Scale Effect}}}_{\text{Effect on Extensive Margin}} \tag{23}$$

The emission tax lowers the total emission level from both its intensive and extensive margin. The presence of a positive emission tax increases the cost of emission, giving incentives for the filled vacancy in sector $d$ to abate the emission level. As a result, the total emission level drops from its intensive margin.

The extensive margin exerts two effects on the total emission level, namely, the substitution effect and the scale effect. The emission tax payment and abatement cost incur a higher flow cost for the filled vacancies in sector $d$, depressing their flow profits and thus the supply of the vacancies in $d$. With a smaller supply of vacancies in $d$, the supply of vacancies, hence the supply of goods, in sector $g$ becomes more abundant than before relative to that in sector $d$. This general equilibrium effect results in a fall in the price $p_g$, thus exerting a pressure for the supply of vacancies in sector $g$ to also decrease. Because the supplies of vacancies in both sectors decline, total vacancies decrease, and the market tightness decreases. Consequently, it becomes harder for the unemployed workers to get a job, thereby lengthening the unemployment spell and increasing the unemployment rate. The overall effect of the emission tax is a shrink in the employment level (the scale effect) and a shrink in the market share of the polluting sector (the substitution effect), both of which reduce the level of emissions in sector $d$. Consequently, the quality of environment improves regardless of the measure. Lemma 2 summarizes the results.

**Lemma 2.** *For all $\tau_x \in \mathbb{R}_{++}$, $\frac{dx^*_{ET}}{d\tau_x} < 0$, $\frac{d\phi^*_{ET}}{d\tau_x} < 0$, $\frac{d\varphi^*_{ET}}{d\tau_x} < 0$, $\frac{de^*_{ET}}{d\tau_x} < 0$, and $\frac{d\chi^*_{ET}}{d\tau_x} < 0$.*

Using Definition 2 and Lemma 2, a proposition follows.

**Proposition 2.** *(Emission Tax) For all $\tau_x \in \mathbb{R}_{++}$, there exists no policy $\mathcal{ET}(\tau_x)$ that is EE preferable to $\varnothing$.*

## 4.2 Emission Standard

We now analyze the impacts of emission standard $\hat{x} \in \mathbb{R}_+$.[14] We denote $\mathcal{EC}(\hat{x})$ as the collection of all emission standards policies with a cap $\hat{x}$, and denote $\Omega_{EC}$ as the set of endogenous variables in the presence of emission cap in a steady state equilibrium. Suppose filled vacancies in sector $d$ are permitted to emit at a level no more than $\hat{x} \geq \bar{x}$. Since the constraint is unbinding, the policy does not affect the economy. That is, $\Omega_{EC} = \Omega_{\varnothing}$.

Consider another case in which filled vacancies in sector $d$ are allowed to emit at a level no more than $\hat{x} < \bar{x}$. That is, the cap is binding. Because abatement incurs a strictly positive cost, filled vacancies have no incentive to abate emission below $x^*_{EC} = \hat{x}$. That is, $x^*_{EC} = \hat{x} < \bar{x} = x^*_{\varnothing}$. Due to the emission constraint, filled vacancies have to pay an abatement cost equal to $c(\bar{x} - \hat{x})$. Since the cost is only applied to vacancies in sector $d$, the revenue differential is equivalent to the abatement cost, and thus

$$\Lambda(\phi^*_{EC}) = \underbrace{c(\bar{x} - \hat{x})}_{\text{Abatement Cost}} \tag{24}$$

Using $\Lambda'(\phi) < 0$, $c'(\cdot) > 0$, $x^*_{EC} = \hat{x}$, equations (17), (19), and (24), and applying Theorem 1, the next lemma follows.

**Lemma 3.** *For all $\hat{x} \in [0, \bar{x})$, $\frac{dx^*_{EC}}{d\hat{x}} > 0$, $\frac{d\phi^*_{EC}}{d\hat{x}} > 0$, $\frac{d\varphi^*_{EC}}{d\hat{x}} > 0$, $\frac{de^*_{EC}}{d\hat{x}} > 0$, and $\frac{d\chi^*_{EC}}{d\hat{x}} > 0$.*

[14] Since the number of workers and the productivity level are fixed in each vacancy, regulating $x$ can be viewed as the regulation on externality or externality per $A$ unit of output. For detailed descriptions of various emission standards in practice, see Holland (2012).

Using Definition 2, Lemma 3 and the assumption that $\hat{x} < \bar{x}$, a proposition follows.

**Proposition 3.** *(Emission Standard) For all $\hat{x} \in [0, \bar{x})$, there exists no policy $\mathcal{EC}(\hat{x})$ that is EE preferable to $\varnothing$.*

## 4.3 Comparison between Emission Tax and Emission Standard

According to Proposition 2 and 3, employment is lower under the two policies, relative to $\varnothing$. Although both policies improve the environment quality, none of them are preferred to $\varnothing$.

Next, we compare the impacts of the two polices. Without loss of generality, fix the emission tax rate $\tau_x \in \mathbb{R}_{++}$, and denote the corresponding set of variables by $\Omega_{ET}$.

First, consider the policy in which the cap is set to $\hat{x} \in [x^*_{ET}(\tau_x), \bar{x})$, where $\tau_x$ is the emission tax rate associated with the optimal emission level $x^*_{ET}(\tau_x)$.[15] Since $\hat{x} \in [x^*_{ET}(\tau_x), \bar{x})$ is binding, Proposition 3 predicts that $x^*_{EC} = \hat{x}$. Hence, $x^*_{EC} \in [x^*_{ET}, \bar{x})$. $x^*_{EC} = x^*_{ET}$ only if $\hat{x} = x^*_{ET}$. Using equations (21) and (24) and $x^*_{EC} \in [x^*_{ET}, \bar{x})$, we have

$$\Lambda(\phi^*_{EC}) = c(\bar{x} - x^*_{EC}) \leq c(\bar{x} - x^*_{ET}) < c(\bar{x} - x^*_{ET}) + \tau_x x^*_{ET} = \Lambda(\phi^*_{ET}).$$

$\Lambda'(\phi) < 0$ and $\Lambda(\phi^*_{EC}) < \Lambda(\phi^*_{ET})$ imply $\phi^*_{EC} > \phi^*_{ET}$ and thus $\varphi^*_{EC} > \varphi^*_{ET}$. Applying Theorem 1, we have $e^*_{EC} > e^*_{ET}$ and thus $\chi^*_{EC} > \chi^*_{ET}$.

Intuitively, vacancies in $d$ are required to pay the same amount of abatement cost under the two policies. But they do not have to pay the emission tax, equal to $\tau_x x^*_{ET}(\tau_x) > 0$, under the emission standard. Therefore, if $\hat{x} = x^*_{ET}(\tau_x)$, the imposition of the emission tax will require vacancies in sector $d$ to pay a higher cost than the cost incurred by emission standard. Therefore, the supply and thus the market share of vacancies in sector $d$ are smaller under the emission tax. That is, $\phi^*_{EC} > \phi^*_{ET}$. If the cap is set at $\hat{x} > x^*_{ET}(\tau_x)$, the required abatement cost is still lower, and the market share of sector $d$ is still higher than that under a cap $\hat{x} = x^*_{ET}(\tau_x)$. Therefore, it is straightforward to see that $\phi^*_{EC} > \phi^*_{ET}$ for all $\hat{x} \in [x^*_{ET}(\tau_x), \bar{x})$.

[15] According to Proposition 2, $x^*_{ET}(\tau_x) < \bar{x}$, $\forall \tau_x \in \mathbb{R}_{++}$.

Next, consider the case where $\hat{x} \in [0, x^*_{ET})$. Before proceeding, we define a critical value $x_\phi(\tau_x)$ as the emission level that solves the following equation:[16]

$$\Lambda(\phi^*_{ET}(\tau_x)) = c(\bar{x} - x_\phi(\tau_x)) \tag{25}$$

where $\phi^*_{ET}(\tau_x)$ is the market share of sector $d$ in the presence of emission tax $\tau_x$. It is not difficult to see that $\phi^*_{EC}(x_\phi) = \phi^*_{ET}(\tau_x)$. That is, $x_\phi(\tau_x)$ is the emission cap such that the market shares of sectors under an emission standard policy with this cap are identical to those under an emission tax policy with emission tax $\tau_x$. Using equation (21), we have

$$c(\bar{x} - x_\phi) = \Lambda(\phi^*_{ET}) = c(\bar{x} - x^*_{ET}) + \tau_x x^*_{ET} > c(\bar{x} - x^*_{ET})$$

Hence, $x_\phi < x^*_{ET}$.

We start with the case in which the government sets the cap to $\hat{x} \in [0, x_\phi]$. Since $\hat{x} \in [0, x_\phi] \subset [0, x^*_{ET}) \subset [0, \bar{x})$, Proposition 3 implies that $x^*_{EC} = \hat{x}$. Using Lemma 3 and equation (25), we have $\phi^*_{EC} \leq \phi^*_{ET}$ $\forall \hat{x} \in [0, x_\phi]$, and $\phi^*_{EC} = \phi^*_{ET}$ only if $\hat{x} = x_\phi$. Therefore, we have $\varphi^*_{EC} < \varphi^*_{ET}$. Theorem 1 implies that $e^*_{EC} \leq e^*_{ET}$ $\forall \hat{x} \in [0, x_\phi]$ and $e^*_{EC} = e^*_{ET}$ only if $\hat{x} = x_\phi$. Since $\hat{x} \leq x_\phi < x^*_{ET}$, $\phi^*_{EC} \leq \phi^*_{ET}$, and $e^*_{EC} \leq e^*_{ET}$, $\chi^*_{EC} < \chi^*_{ET}$ $\forall \hat{x} \in [0, x_\phi]$.

We are left with the case $\hat{x} \in (x_\phi, x^*_{ET})$. Following a similar argument, it is straightforward to show that $x^*_{EC} = \hat{x} \in (x_\phi, x^*_{ET})$, $\phi^*_{EC} > \phi^*_{ET}$, $e^*_{EC} > e^*_{ET}$ for all $\hat{x} \in (x_\phi, x^*_{ET})$. The following proposition summarizes the results.

**Proposition 4.** *(Comparison between Emission Tax and Emission Standard) Suppose a government sets an effective emission cap $\hat{x} \in [0, \bar{x})$. Then, $x^*_{EC} = \hat{x}$. In addition,*

- *For all $\hat{x} \in [0, x_\phi]$, $x^*_{EC} < x^*_{ET}$, $\phi^*_{EC} \leq \phi^*_{ET}$, $\varphi^*_{EC} < \varphi^*_{ET}$, $e^*_{EC} \leq e^*_{ET}$, and $\chi^*_{EC} < \chi^*_{ET}$; and*
- *For all $\hat{x} \in [x^*_{ET}, \bar{x})$, $x^*_{EC} \geq x^*_{ET}$, $\phi^*_{EC} > \phi^*_{ET}$, $\varphi^*_{EC} > \varphi^*_{ET}$, $e^*_{EC} > e^*_{ET}$ and $\chi^*_{EC} > \chi^*_{ET}$.*

[16] $x_\phi(\tau_x)$ is dependent on $\tau_x$.

*where $x_\phi$ solves equation (25). Furthermore, there exists a unique critical emission standard $x^c \in (x_\phi, x^*_{ET})$ such that*

- *For all $\hat{x} \in (x_\phi, x^c)$, $x^*_{EC} < x^*_{ET}$, $\phi^*_{EC} > \phi^*_{ET}$, $\varphi^*_{EC} < \varphi^*_{ET}$ and $e^*_{EC} > e^*_{ET}$;*
- *For all $\hat{x} \in [x^c, x^*_{ET})$, $x^*_{EC} < x^*_{ET}$, $\phi^*_{EC} > \phi^*_{ET}$, $\varphi^*_{EC} \geq \varphi^*_{ET}$ and $e^*_{EC} > e^*_{ET}$.*

*Proof.* See Appendix. □

Using Definition 2, Proposition 4, and the assumption that $\hat{x} < \bar{x}$, a theorem follows.

**Theorem 3.** ***(Unpreferred Emission Tax Theorem)*** *For all $\tau_x, \hat{x} \in \mathbb{R}_{++}$, there exists no $\mathcal{ET}(\tau_x)$ such that $\mathcal{ET}(\tau_x)$ is EE preferable to $\mathcal{EC}(\hat{x})$. In addition, for all $\tau_x \in \mathbb{R}_{++}$, there always exists a corresponding interval $\tilde{x}(\tau_x) \in (x_\phi(\tau_x), x^c(\tau_x))$ such that for all $\hat{x} = \tilde{x}$, $\mathcal{EC}(\hat{x})$ is EE preferable to $\mathcal{ET}(\tau_x)$, where $\phi^*_{EC}(x_\phi) = \phi^*_{ET}(\tau_x)$ and $x_\phi(\tau_x) < x^c < x^*_{ET}(\tau_x)$.*

Theorem 3 predicts that if employment and environment quality are the only variables of interest, there always exist some emission caps under which an emission standard policy is preferred to an emission tax policy. That is, for any emission tax rate of an emission tax policy, one can always find an emission cap under which the emission standard policy leads to a higher level of employment and a lower emission per output than the taxation scheme. The policy implication is that the public and policymakers should not introduce emission tax whenever they can use emission standard policies.

It is noteworthy that the above result relies on one important assumption: the use of the emission tax revenue is irrelevant in the policy selection process. The next section will investigate the case where an emission tax policy is used, and the tax revenue is redistributed to subsidize filled vacancies.

# 5 Revenue-Neutral Emission Taxes and Other Policies

In the past few decades, the literature suggests that the environmental tax revenue could be redistributed to subsidize filled vacancies. The motivation of this suggestion is that such redistribution scheme can improve the environment and simultaneously reduce the distortion cost of taxation. This is the "double dividend hypothesis".[17] Although normative analysis is not the focus of this paper, inspired by the idea of double dividend hypothesis, this section investigates the situation in which the emission tax revenue is recycled to subsidize filled vacancies.

Denote $\mathcal{RN}(\tau_x)$ as a revenue-neutral emission taxation scheme with the tax revenue equally redistributed to all filled vacancies in a lump-sum fashion.[18] Also denote $\Omega_{RN}$ as the corresponding set of variables of our interest under this policy.

Suppose a government subsidizes $h_j$ to each filled vacancy in sector $j$ in a lump-sum fashion. Taking $h_j$ as given, the asset values of filled vacancies are

$$\begin{aligned} rJ_d^F &= \max_x \left\{p_d A - w_d(1+\tau_w) - c(\bar{x}-x) - \tau_x x + \lambda(J_d^V - J_d^F) + h_d\right\} \\ rJ_g^F &= p_g A - w_g + \lambda(J_g^V - J_g^F) + h_g \end{aligned} \tag{26}$$

With a wage tax, the sharing rule is identical to the rule (8). Using equations (3), (8), and (26), the wages are given by

$$\begin{aligned} (1+\tau_w)w_d &= (1+\tau_w)(1-\beta)rJ^U + \beta\left(p_d A - c(\bar{x}-x^*) - \tau_x x^* - rk + h_d\right) \\ (1+\tau_w)w_g &= (1+\tau_w)(1-\beta)rJ^U + \beta\left(p_g A - rk + h_g\right) \end{aligned} \tag{27}$$

Using equations (7), (8), (9), and (27), the two zero-profits conditions are given by

$$\begin{aligned} rk &= \frac{q(\theta)(1-\beta)}{r+\lambda}\left(p_d(\phi)A - c(\bar{x}-x^*) - \tau_x x^* - (1+\tau_w)z - rk\left(1+\frac{(1+\tau_w)\theta\beta}{1-\beta}\right) + h_d\right) \\ rk &= \frac{q(\theta)(1-\beta)}{r+\lambda}\left(p_g(\phi)A - (1+\tau_w)z - rk\left(1+\frac{(1+\tau_w)\theta\beta}{1-\beta}\right) + h_g\right) \end{aligned} \tag{28}$$

[17] See Goulder (1995), Parry (1995), Parry and Bento (2000), Bento and Jacobsen (2007), Manresa and Sancho (2005), Chiroleu-Assouline and Fodha (2006), Dissou and Sun (2013), and Williams et al. (2015).

[18] Most of the results in this section remain unchanged if the tax revenue is redistributed in a lump-sum fashion, or via a reduction in corporate taxes and/or wage taxes.

In a steady state equilibrium, the total emission tax revenue is equal to $\tau_x x^* e^* \phi^*$. Suppose that the total emission revenue is recycled to subsidize all filled vacancies regardless of sector type. Then, $h_d = h_g = \tau_x x^* e^* \phi^* / e^* = \tau_x x^* \phi^*$. Taking $h_j$ as given, the optimal emission level $x^*_{RN}$ is unaffected and solves the first order condition (20). That is, $x^*_{RN} = x^*_{ET}$. Since the revenue is equally distributed amongst filled vacancies, the redistribution of the tax revenue does not alter the revenue differential between the two sectors, and the revenue differential is thus expressed in the same way as (21). So, the market share of each sector type is unaffected. That is, $\phi^*_{RN} = \phi^*_{ET}$. Since the redistribution of the tax revenue does not alter the intensive margin of emission or the substitution effect, recycling the tax revenue does not affect the emission per output. With the transfer, the flow profits rise; hence, the supplies of vacancies of both types increase. Consequently, $e^*_{RN} > e^*_{ET}$, and $\chi^*_{RN} > \chi^*_{ET}$. The higher level of the total emission under $\mathcal{RN}(\tau_x)$ is driven by the rise of the employment level. But $\varphi^*_{RN} = \varphi^*_{ET}$ because of equation (19).

Next, we compare between $\Omega_{RN}$ and $\Omega_{\varnothing}$. According to Proposition 2, $x^*_{RN} = x^*_{ET} < x^*_{\varnothing}$ because the lump-sum transfer does not affect the emission decision on the intensive margin but the introduction of the emission tax lowers the emission per vacancy. Similarly, we have $\phi^*_{RN} = \phi^*_{ET} < \phi^*_{\varnothing}$. While the transfer does not alter the revenue differential between two sectors, the emission tax increases the differential. As a result, the market share of polluting sector is lower under $\mathcal{RN}(\tau_x)$ than under $\varnothing$. Summarizing these predictions and applying Proposition 2 yield the following proposition.

**Proposition 5.** *Suppose a government introduces an emission tax $\tau_x$ and equally rebates the tax revenue to filled vacancies in a lump-sum transfer. Then,*

- $x^*_{RN} = x^*_{ET}$, $\phi^*_{RN} = \phi^*_{ET}$, $e^*_{RN} > e^*_{ET}$, $\varphi^*_{RN} = \varphi^*_{ET}$, *and* $\chi^*_{RN} > \chi^*_{ET}$*;*
- $x^*_{RN} < x^*_{\varnothing}$, *and* $\phi^*_{RN} < \phi^*_{\varnothing}$.

According to this proposition, $\mathcal{RN}(\tau_x)$ is EE preferable to $\mathcal{ET}(\tau_x)$. Under $\mathcal{RN}(\tau_x)$, the emission per output is lower for two reasons. First, the emission tax increases the cost of emission, decreasing the intensive margin of emission. Second, the increased cost generates the revenue differential, thereby decreasing the market share of polluting sector in the steady state equilibrium. These two forces lower the emission per output.

Nevertheless, we are uncertain whether $\mathcal{RN}(\tau_x)$ is preferred to $\varnothing$ or not. The following theorem states that under certain circumstances, there always exists a $\tau_x \in \mathbb{R}_{++}$ such that employment is larger under $\mathcal{RN}(\tau_x)$ than under $\varnothing$.

**Theorem 4.** ***(Revenue-Neutral Emission Tax)***

1. *For all $\tau_x \in \mathbb{R}_{++}$, $\mathcal{RN}(\tau_x)$ is EE preferable to $\mathcal{ET}(\tau_x)$.*

2. *If the following condition (29) is satisfied, there exists at least one $\tau_x \in \mathbb{R}_{++}$ such that $\mathcal{RN}(\tau_x)$ is EE preferable to $\varnothing$.*

$$\frac{(1-\gamma)|\varepsilon_{p_g,\tau_x}|}{\gamma|\varepsilon_{p_d,\tau_x}| + (1-\gamma)|\varepsilon_{p_g,\tau_x}|} \quad \leq \quad \alpha \;\; or \;\; \frac{\gamma|\varepsilon_{p_d,\tau_x}|}{\gamma|\varepsilon_{p_d,\tau_x}| + (1-\gamma)|\varepsilon_{p_g,\tau_x}|} \geq 1-\alpha \tag{29}$$

   *where $\gamma \equiv \left.\frac{\phi^\rho}{\phi^\rho+(1-\phi)^\rho}\right|_{\phi=\phi^*_\varnothing}$ and $\varepsilon_{p_j,\tau_x} \equiv \left.\frac{dp_j}{d\tau_x}\frac{\tau_x}{p_j}\right|_{\tau_x=0}$ is the price elasticity of good $j$ with respect to $\tau_x$ evaluated at $\tau_x = 0$.*

3. *If the condition (29) is satisfied, $\mathcal{RN}(\tau_x)$ is EE preferable to $\mathcal{EC}(\hat{x})$ for all $\tau_x \in \Theta$ and $\hat{x} \in [x^*_{ET}, \bar{x}]$, where $\Theta$ is the collection of $\tau_x$ such that $\mathcal{RN}(\tau_x)$ is EE preferable to $\varnothing$. Furthermore, $\mathcal{RN}(\tau_x)$ is not EE preferable to $\mathcal{EC}(\hat{x})$, and $\mathcal{EC}(\hat{x})$ is not EE preferable to $\mathcal{RN}(\tau_x)$ for all $\tau_x \in \mathbb{R}_{++}$ and $\hat{x} \in [0, x_\phi]$.*

*Proof.* See Appendix. □

Intuitively, $\phi^*$ falls because of the introduction of the emission tax. Therefore, the clean goods become relatively more abundant. This general equilibrium effect reduces the relative price of clean goods, depressing their vacancy supply. Hence, employment declines in the presence of emission tax. On the other hand, the transfer to filled vacancies induces the supply of vacancies in both sectors, causing the employment rate to rise. Therefore, whether employment increases under $\mathcal{RN}(\tau_x)$ relative to null policy is inconclusive. The condition (29) ensures that the responsiveness of the price of clean goods with respect to the emission tax is low. As such, the decline in employment from the general equilibrium effect is small enough and is outweighed by the rise in employment from the direct effect of the transfer on the vacancies' flow profit. Hence, if the condition (29) is satisfied, $\mathcal{RN}(\tau_x)$ is EE preferable to $\varnothing$.

# 6 Conclusion

This paper constructs a two-sector model that endogenizes frictional unemployment, and both the extensive and the intensive margin of pollutant emission. The model is an analytically tractable tool and can be easily extended to analyze the economic consequence of various commonly seen environmental policies on labor market outcomes and the quality of the environment. Several interesting analytical results emerge as follows.

The main contribution to the literature is the theorem of impossibility to avoid tradeoff between employment and the extensive margin of emission under policies that only regulate the polluting sector and do not decrease the intensive margin of emission. In the spirit of this result, our model shows that a number of environmental policies are not Employment and Environment (EE) preferable to no intervention. For example, there exists no policy that regulates the polluting sector only, does not alter the intensive margin of emission, and is EE preferable to no intervention. Therefore, it is impossible to impose any corporate tax, wage tax, capital tax, and/or lump sum tax in the polluting sector and yield an equilibrium that is EE preferable to a laissez-faire equilibrium. These policies will lead to either lower employment level or poorer environment quality. Furthermore, this paper indicates that if policymakers have to select either emission standards or linear emission taxes, there always exists an interval of emission standards in which the emission standard is EE preferable to linear emission taxes for all positive tax rates.

Second, this paper is the first theoretical paper to demonstrate that a revenue-neutral emission tax could be EE preferable to no intervention. In other words, we demonstrate that under a condition there always exists an emission tax rate under which the revenue-neutral emission taxation scheme improves the environment quality and simultaneously increases employment. On the one hand, it provides sufficient incentive for a decentralized economy to cut emission, correcting the distortion generated by externality. More importantly, by recycling the tax revenue to subsidize job opening, the policy is shown to increase employment, thereby boosting the economy and easing the public concern of job loss. On the other hand, it does not increase the fiscal burden because of its nature of revenue-neutrality. In addition, a revenue-neutral emission tax is shown to be EE preferable to linear emission taxes for all positive tax rates, and also EE preferable to emission standards over certain intervals of emission caps. Hence, this paper provides theoretical support for revenue-neutral emission taxes and the double dividend hypothesis when involuntary unemployment on the equilibrium is considered.

# 7 Appendix: Proof

## 7.1 Proof of Proposition 1

When $x^*$ tends to zero ($\bar{x}$), the LHS approaches positive infinity (zero) in equation (16). For all $x^* \in [0, \bar{x}]$, the RHS is nonnegative finite. The intermediate value theorem ensures that there exists $x^* \in (0, \bar{x})$ such that equation (6) is satisfied. Since $-c''(\bar{x} - x^*) + S''(\bar{x} - x^*) < 0$ and $\tau_x''(x^*)x^* + 2\tau_x'(x^*) \geq 0$ for all $x_d^* \in (0, \bar{x})$, there exists a unique $x^*$ such that equation (6) is satisfied.

Rewrite the revenue differential (16), we have

$$\Lambda(\phi) = c(\bar{x} - x^*) - S(\bar{x} - x^*) + \tau_x(x^*)x^*$$

where the unique $x^*$ is pinned down by equation (6). Hence, the RHS of the above equation is positive and independent of $\phi$. When $\phi$ approaches zero (one), the LFS tends to positive (negative) infinity. The intermediate value theorem ensures that there exists $\phi^* \in (0,1)$ such that the above equation is satisfied. Since the LHS is strictly decreasing in $\phi$, the $\phi^*$ is unique. When $\theta$ tends to zero, the RHS of equation (15) approaches positive infinity because of $\lim_{\theta\to 0} q(\theta) = \infty$. When $\theta$ tends to positive infinity, its RHS approaches negative infinity because of $\lim_{\theta\to\infty} q(\theta) = 0$ and $\lim_{\theta\to\infty} \theta q(\theta) = \infty$. As the LHS is a constant, the intermediate value theorem ensures that there exists $\theta^*$ such that equation (15) is satisfied. Since the RHS is strictly decreasing in $\theta$, $\theta^*$ is unique. Substituting the unique $\theta^*$ into equation (10) yields the unique steady state $e^*$ and $u^*$.

## 7.2 Proof of Lemma 1

Suppose there are two policies $\mathcal{P}$ and $\mathcal{Q}$ such that $\mathcal{P}$ is strictly preferred to $\mathcal{Q}$. Then, either $x_\mathcal{P}\phi_\mathcal{P}e_\mathcal{P} < x_\mathcal{Q}\phi_\mathcal{Q}e_\mathcal{Q}$ and $e_\mathcal{P} \geq e_\mathcal{Q}$ or $x_\mathcal{P}\phi_\mathcal{P}e_\mathcal{P} \leq x_\mathcal{Q}\phi_\mathcal{Q}e_\mathcal{Q}$ and $e_\mathcal{P} > e_\mathcal{Q}$. In the first case, we have

$$x_\mathcal{P}\phi_\mathcal{P} < x_\mathcal{Q}\phi_\mathcal{Q}\frac{e_\mathcal{Q}}{e_\mathcal{P}} \leq x_\mathcal{Q}\phi_\mathcal{Q}$$

The last inequality uses the result $e_\mathcal{P} \geq e_\mathcal{Q}$ and thus $\frac{e_\mathcal{Q}}{e_\mathcal{P}} \leq 1$. Hence, $e_\mathcal{P} \geq e_\mathcal{Q}$ and $x_\mathcal{P}\phi_\mathcal{P} < x_\mathcal{Q}\phi_\mathcal{Q}$. In the second case, we have

$$x_\mathcal{P}\phi_\mathcal{P} \leq x_\mathcal{Q}\phi_\mathcal{Q}\frac{e_\mathcal{Q}}{e_\mathcal{P}} < x_\mathcal{Q}\phi_\mathcal{Q}$$

Hence, $e_\mathcal{P} > e_\mathcal{Q}$ and $x_\mathcal{P}\phi_\mathcal{P} < x_\mathcal{Q}\phi_\mathcal{Q}$. The proof is completed.

## 7.3 Proof of Proposition 4

First, we show that $\varphi(x_\phi, \phi^*_{ET}) < \varphi(x^*_{ET}, \phi^*_{ET}) < \varphi(x^*_{ET}, \phi^*_{EC}(x^*_{ET}))$, where $\phi^*_{EC}(x^*_{ET})$ solves $\Lambda(\phi) = c(\bar{x} - x^*_{ET})$. Since $x_\phi < x^*_{ET}$ and Lemma 3 shows that $d\varphi/d\hat{x} > 0$, we have $\varphi(x_\phi, \phi^*_{ET}) < \varphi(x^*_{ET}, \phi^*_{ET})$.

Using equations (21) and (24), we have

$$\Lambda(\phi_{ET}^*) = c(\bar{x} - x_{ET}^*) + \tau_x x_{ET}^* > c(\bar{x} - x_{ET}^*) = \Lambda(\phi_{EC}^*(x_{ET}^*))$$

$\Lambda'(\phi) < 0$ and $\Lambda(\phi_{ET}^*) > \Lambda(\phi_{EC}^*(x_{ET}^*))$ imply $\phi_{ET}^* < \phi_{EC}^*(x_{ET}^*)$. Applying Lemma 3 yields $\varphi(x_{ET}^*, \phi_{ET}^*) < \varphi(x_{ET}^*, \phi_{EC}^*(x_{ET}^*))$.

Now we have shown that $\varphi(x_\phi, \phi_{EC}^*(x_\phi)) < \varphi(x_{ET}^*, \phi_{ET}^*) < \varphi(x_{ET}^*, \phi_{EC}^*(x_{ET}^*))$, where $\phi_{EC}^*(x_\phi) \equiv \phi_{ET}^*$. Note that $\varphi(\hat{x}, \phi_{EC}^*(\hat{x}))$ is continuous in $\hat{x}$ and $\phi_{EC}^*(\hat{x})$, and $\phi_{EC}^*(\hat{x})$ is continuous in $\hat{x}$. The intermediate value theorem ensures that there exists $x^c \in (x_\phi, x_{ET}^*)$ such that $\varphi(x^c, \phi_{EC}^*(x^c)) = \varphi(x_{ET}^*, \phi_{ET}^*)$. Since Lemma 3 indicates that $d\varphi/d\hat{x} > 0$, we have $x^c$ is unique, $\varphi_{EC}^* < \varphi_{ET}^*$ for all $\hat{x} \in (x_\phi, x^c)$ and $\varphi_{EC}^* \geq \varphi_{ET}^*$ for all $\hat{x} \in [x^c, x_{ET}^*)$. This completes the proof.

## 7.4 Proof of Theorem 4

**The First Statement.** The first statement follows immediately from Proposition 5 and Definition 2.

**The Second Statement.** First, using equations (10) and (15), it is straightforward to show that

$$\frac{d\left(\Pi_{RN,g} - \frac{\theta\beta rk}{1+\beta}\right)}{d\tau_x} \gtreqless 0 \text{ iff } \frac{de}{d\tau_x} \gtreqless 0$$

Next, we show that $d\left(\Pi_{RN,g} - \frac{\theta\beta rk}{1+\beta}\right)/d\tau_x$ and $d\Pi_{RN,g}/d\tau_x$ have the same sign.

$$\frac{d\left(\Pi_{RN,g} - \frac{\theta\beta rk}{1+\beta}\right)}{d\tau_x} = \frac{d\Pi_{RN,g}}{d\tau_x} - \frac{\beta rk}{1+\beta}\frac{d\theta}{d\tau_x}$$

Using equation (15), one can show that the signs of the left hand side of the above equation and

$d\theta/d\tau_x$ are identical. Rearranging terms, we have

$$\frac{d\left(\Pi_{RN,g} - \frac{\theta\beta rk}{1+\beta}\right)}{d\tau_x} + \frac{\beta rk}{1+\beta}\frac{d\theta}{d\tau_x} = \frac{d\Pi_{RN,g}}{d\tau_x}$$

We are done with the proof that $d\left(\Pi_{RN,g} - \frac{\theta\beta rk}{1+\beta}\right)/d\tau_x$ and $d\Pi_{RN,g}/d\tau_x$ have the same sign. Hence, it is sufficient to pin down the condition such that $d\Pi_{RN,g}/d\tau_x \gtreqless 0$. Using equation (24), we have

$$\Pi_{RN,g} = p_g(\phi)A - rk - (1+\tau_w)z + \phi\left(p_d(\phi)A - p_g(\phi)A - C(\bar{x}-x)\right)$$

Differentiating $\Pi_{RN,g}$ with respect to $\tau_x$ and using Lemma 2, we have

$$\frac{d\Pi_{RN,g}}{d\tau_x} = \underbrace{(1-\phi)A\frac{dp_g(\phi)}{d\tau_x}}_{\text{-ve}} + \phi\left(\underbrace{A\frac{dp_d(\phi)}{d\tau_x}}_{\text{+ve}} - \underbrace{\frac{dC(\bar{x}-x)}{d\tau_x}}_{\text{+ve}}\right) + \underbrace{\tau_x x\frac{d\phi}{d\tau_x}}_{\text{-ve}}$$

Taking the limit $\tau_x \to 0$, we have

$$\lim_{\tau_x\to 0}\frac{d\Pi_{RN,g}}{d\tau_x} = \lim_{\tau_x\to 0}(1-\phi)A\frac{dp_g(\phi)}{d\tau_x} + \lim_{\tau_x\to 0}\phi A\frac{dp_d(\phi)}{d\tau_x} - \lim_{\tau_x\to 0}\frac{dC(\bar{x}-x)}{d\tau_x} + \lim_{\tau_x\to 0}\tau_x x\frac{d\phi}{d\tau_x}$$

Note that

$$\lim_{\tau_x\to 0}\frac{dC(\bar{x}-x)}{d\tau_x} = \lim_{\tau_x\to 0}\frac{dC(\bar{x}-x)}{dx}\frac{dx}{d\tau_x} = 0 \text{ and } \lim_{\tau_x\to 0}\tau_x x\frac{d\phi}{d\tau_x} = 0$$

When $\tau_x$ tends to zero, $x$ approaches $\bar{x}$. Since $\lim_{x\to\bar{x}}\frac{dC(\bar{x}-x)}{dx} = 0$ and $|\frac{dx}{d\tau_x}| \leq \bar{x} < \infty$, $\lim_{\tau_x\to 0}\frac{dC(\bar{x}-x)}{d\tau_x} = 0$. Since $|\frac{d\phi}{d\tau_x}| \leq 1 < \infty$, $\lim_{\tau_x\to 0}\tau_x x\frac{d\phi}{d\tau_x} = 0$. Hence, when $\tau_x$ tends to zero,

$$\frac{d\Pi_{RN,g}}{d\tau_x} = (1-\phi)A\frac{dp_g(\phi)}{d\tau_x} + \phi A\frac{dp_d(\phi)}{d\tau_x}$$

So, $\frac{d\Pi_{RN,g}}{d\tau_x} \gtreqless 0$ iff

$$\begin{aligned}
\phi A \frac{dp_d(\phi)}{d\tau_x} &\gtreqless -(1-\phi) A \frac{dp_g(\phi)}{d\tau_x} \\
\phi \varepsilon_{p_d,\tau_x} \frac{p_d}{p_g} &\gtreqless -(1-\phi)\varepsilon_{p_g,\tau_x}
\end{aligned}$$

Using equation (11), we have

$$\begin{aligned}
\varepsilon_{p_d,\tau_x} \frac{\alpha}{1-\alpha} \left(\frac{\phi}{1-\phi}\right)^{\rho} &\gtreqless -\varepsilon_{p_g,\tau_x} \\
\alpha \phi^{\rho} \varepsilon_{p_d,\tau_x} &\gtreqless -(1-\alpha)(1-\phi)^{\rho} \varepsilon_{p_g,\tau_x} \\
\alpha \left(\phi^{\rho}\varepsilon_{p_d,\tau_x} - (1-\phi)^{\rho}\varepsilon_{p_g,\tau_x}\right) &\gtreqless -(1-\phi)^{\rho}\varepsilon_{p_g,\tau_x} \\
\alpha &\gtreqless \frac{-\gamma \varepsilon_{p_g,\tau_x}}{(1-\gamma)\varepsilon_{p_d,\tau_x} - \gamma \varepsilon_{p_g,\tau_x}}
\end{aligned}$$

So, if $\alpha \geq \frac{(1-\gamma)\varepsilon_{p_g,\tau_x}}{\gamma \varepsilon_{p_d,\tau_x}(1-\gamma)\varepsilon_{p_g,\tau_x}}$, there exists at least one $\tau_x \in \mathbb{R}_{++}$ such that $e_{RN} \geq e_{\varnothing}$. Using $\varphi_{RN} < \varphi_{\varnothing}$ in Proposition 5, the proof of the second statement is completed.

**The Third Statement.** First, suppose $\hat{x} = \bar{x}$, $e_{EC} = e_{\varnothing}$. Lemma 3 implies that $e_{EC} < e_{\varnothing}$ for all $\hat{x} = [0, \bar{x})$. Since the condition (29) is satisfied, the second statement implies for all $\tau_x \in \Theta$ $e_{RN} > e_{EC}$ for all $\hat{x} = [0, \bar{x})$, and $\Theta$ is nonempty. Proposition 4 and 5 imply that given $\tau_x \in \mathbb{R}_{++}$, $\varphi_{RN} < \varphi_{EC}$ for all $\hat{x} \in [x^*_{ET}, \bar{x})$.

Next, given $\tau_x \in \mathbb{R}_{++}$, Proposition 4 and 5 imply that $\varphi_{EC} < \varphi_{RN}$ and $e_{EC} \leq e_{RN}$ for all $\hat{x} \in [0, x_{\phi}]$. By Definition 2, $\mathcal{RN}(\tau_x)$ is not preferred to $\mathcal{EC}(\hat{x})$, and $\mathcal{EC}(\hat{x})$ is not preferred to $\mathcal{RN}(\tau_x)$ for all $\tau_x \in \mathbb{R}_{++}$ and $\hat{x} \in [0, x_{\phi}]$. This completes the proof.

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