# ONLINE APPENDIX: ON THE AMBIGUITY OF JOB SEARCH

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# Contents



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# 1 Appendix A: Comparison with Nishimura and Ozaki (2004)

This appendix compares our model with the one under Knightian uncertainty (Nishimura and Ozaki, 2004). We only considers a worker's problem as in (Nishimura and Ozaki, 2004). In this model, workers do not know the data-generating process of an economy. Therefore, they utilize an approximating model that best describes the data-generating process. Meanwhile, they fear possible misspecification of the approximating model. Consequently, they do not make decisions simply according to the rational expectation paradigm. Instead, they incorporate a penalty function into their value function to diverge from the approximating model. They distort the approximating model so that the fear of model misspecification can be considered. In addition to the decisions on job searches as in the conventional search and matching model, the model of choices is also endogenized. Following the tradition in the literature, we call the optimal model chosen by a robust agent a distorted model.

Consider a discrete time economy with infinitely lived workers who are ambiguity-averse. Denote $\beta = 1/(1+r)$ as a discounted factor, where $r$ is a real interest rate. A worker is either unemployed or employed. An unemployed worker enjoys an unemployment benefit $b$ and receives a job offer at the end of each period. When an unemployed worker gets a job offer, a match-specific productivity level $\delta$ is realized, where the cumulative distribution function of $\delta$ is given by $F(\delta)$, and the corresponding probability density function is denoted by $f(\delta)$. Denote $J^E(\delta)$ and $J^U$ as discounted present values of employment and unemployment, respectively. An unemployed worker accepts a job offer only if $J^E(\delta)$ is at least as large as his/her outside option value $J^U$.

Nevertheless, an unemployed worker may not be strongly confident about the specification of a productivity distribution $F(\delta)$. Meanwhile, the unemployed worker prefers a known productivity distribution; as a result, an unemployed worker maximizes the minimum expected outcome as follows:

$$J^U = b + \beta \min_{m(x)} \left( \mathbb{E}_x \left( m(x) \max\{J^E(x), J^U\} \right) - \frac{1}{\alpha} \mathbb{E}_x m(x) \ln m(x) \right) \tag{1}$$

subject to $\int m(x) dF(x) = 1$. The unemployed worker chooses $m(\delta) = \hat{f}(\delta)/f(\delta)$, a likelihood ratio, to minimize the last term of the preceding equation. $Em \ln m$, known as a relative entropy, measures the Kullback-Leibler distance between two distributions. In the present model, it measures the discrepancy between the approximating and the chosen model. $\alpha \leq 0$ is a penalty parameter for the relative entropy (the last term of the preceding equation), which captures the degree of ambiguity aversion of workers. The more the workers fear about the model misspecification, the lower $\alpha$ will be. The optimal likelihood ratio that satisfies $\int m(x) dF(x) = 1$ is given by

$$\frac{\hat{f}(\delta)}{f(\delta)} = \frac{e^{\alpha \max\{J^E(\delta), J^U\}}}{\int_0^\infty e^{\alpha \max\{J^E(x), J^U\}} dF(x)}. \tag{2}$$

$\alpha = 0$ implies that $\hat{f}(\delta) = f(\delta)$ and thus $J^U = b + \beta \mathbb{E}_x \left(\max\{J^E(x), J^U\}\right)$, which is the case in the absence of the fear of model misspecification. When a worker does not worry about model misspecification, his/her belief of the probability density function is identical to the approximating one. Hence, the conventional search and matching model is a special case of ours when $\alpha$ equals zero.

We make an assumption that an employed worker is paid with an output $\delta$. A worker, once employed, will stick to the job forever. Hence, $J^E(\delta) = \delta/(1-\beta)$, and $\partial J^E(\delta)/\partial\delta = 1/(1-\beta)$. This positive partial derivative implies that the likelihood ratio $m(\delta)$ declines with $\delta$. Interestingly, this result implies that to minimize an expected outcome an unemployed worker chooses to believe that higher match-specific productivity levels are less likely to be realized.

When $\delta$ goes to zero, $J^E = 0$ is less than $J^U$. Together with the fact that $\partial J^E(\delta)/\partial\delta > 0$, there exists a unique reservation productivity threshold $\delta^R$, above which workers will accept the job offer. Hence, we have $\delta^R = (1-\beta)J^U$. Substituting the reservation productivity level and the optimal likelihood ratio (2) into equation (1), simple algebra yields

$$\frac{r\delta^R}{1+r} \quad = \quad b + \frac{\beta}{\alpha} \ln \left( F(\delta^R) + \int_{\delta^R}^\infty e^{\alpha(x-\delta^R)} dF(x) \right)$$

Partially differentiating the preceding equation with respect to $\alpha$, it is straightforward to show that the higher $\alpha$, the higher will be the threshold $\delta^R$. Nishimura and Ozaki (2004) build a similar model to demonstrate the impact of Knightian uncertainty on the reservation productivity threshold.[1] In Nishimura and Ozaki (2004), an unemployed worker, under Knightian uncertainty, minimizes his/her expected discounted future income as follows:

$$J^U = \min_{p \in \mathcal{P}_0} \left\{ \int_W I(w) dP(w) \right\}, \tag{3}$$

where $I(w)$, a discounted future income, is a bounded measurable function of the observed offer $w$. The formula of $I(w)$ is given by equation (18) of their paper. $\mathcal{P}_0$ is a set of distributions; Knightian uncertainty increases with the size of the set $\mathcal{P}_0$. The key difference between Nishimura and Ozaki (2004) and our model lies on the specification of the probability set $\mathcal{P}_0$. In Nishimura and Ozaki (2004), the $\mathcal{P}_0$ in the multiple prior expected utility is unrestricted.[2] In the present model, the size

[1] In section 2 of Nishimura and Ozaki (2004), a similar model is built for demonstration, followed by a more rigorous analysis under general preference and productivity distribution in section 4.

[2] Since the current paper does not aim to investigate Knightian uncertainty, readers interested in its axiomatic

of the probability set depends on both the reference density $f(\delta)$ and the penalty index $\alpha$. In fact, according to Hansen and Sargent (2001), the multiplier problem (1) is equivalent to the following constraint problem:

$$\min_{\hat{F}}\left\{\int_W I(w)d\hat{F}(w)\right\} \text{subject to } \mathbb{E}_{\hat{F}}[\ln(d\hat{F}/dF)] < \gamma, \tag{4}$$

which is equivalent to the problem (3) with $\mathcal{P}_0$ restricted to the set of the probability distribution $\hat{F}$ that satisfies $\mathbb{E}_{\hat{F}}[\ln(d\hat{F}/dF)] < \gamma$.[3] $\gamma$ is an upper bound of the discrepancy between the approximated distribution $\hat{F}$ and the prior distribution $F$, it depends on the penalty index $\alpha$. The lower $\alpha$, the higher will be $\gamma$; thus, a lower $\alpha$ implies that a larger set of the distribution $\hat{F}$ can be chosen. Therefore, the use of $\alpha$ as a measure of ambiguity aversion is in line with Nishimura and Ozaki (2004). In the extreme case where $\alpha \to 0$, we have $\gamma \to 0$; as a result, only the prior distribution $F$ satisfies $\mathbb{E}_{\hat{F}}[\ln(d\hat{F}/dF)] < \gamma$. This is equivalent to the extreme case in Nishimura and Ozaki (2004), where the size of the probability set reduces to a singleton $\mathcal{P}_0 = \{F\}$.[4]

# 2 Appendix B: The DMP Model with Risk Preferences

This appendix demonstrates the differences in the implications of ambiguity and risk preferences in the DMP model. Workers are assumed to be risk-averse and ambiguity-neutral, and vacancies are assumed to be risk-neutral and ambiguity-neutral. We assume a worker has a CRRA utility function $u(x) = \frac{x^{1-\varepsilon}}{1-\varepsilon}$, where $\varepsilon > 0$ is the parameter of risk aversion. $x$ measures a worker's income; hence, $x$ equals an unemployment benefit $b$ for unemployed workers and equals a wage $w(\delta)$ for employed workers with a productivity level $\delta$.

The value function of employment with productivity $\delta$ is given by

$$J^E(\delta) = \frac{w(\delta)^{1-\varepsilon}}{1-\varepsilon} + \beta\Big[(1-\lambda)J^E(\delta) + \lambda J^U\Big].$$

Similarly, the value function of unemployment is given by

$$J^U = \frac{b^{1-\varepsilon}}{1-\varepsilon} + \beta\Big[\theta q(\theta)\mathbb{E}_x(\max\{J^E(x), J^U\}) + (1-\theta q(\theta))J^U\Big].$$

foundation are referred to Gilboa and Schmeidler (1989).

[3]Readers who are interested in its application to financial markets are referred to Cao et al. (2005).

[4]While the multiple-prior expected utility in Nishimura and Ozaki (2004) is not nested by the multiplier utility in our model, Werner (2011) points out that the multiple-prior preference is in the class of variational preference where cost function is given by $C_{\mathcal{P}_0}\{P\} = \begin{cases} 0 & \text{if } P \in \mathcal{P}_0 \\ +\infty & \text{if } P \notin \mathcal{P}_0 \end{cases}$.

The value function of filled vacancies with productivity $\delta$ is given by

$$J^F(\delta) = \delta - w(\delta) + \beta\Big[(1-\lambda)J^F(\delta) + \lambda J^V\Big].$$

Similarly, the value function of unfilled vacancies is given by

$$J^V = -c + \beta\Big[q(\theta)\mathbb{E}_x \max\{J^F(x), J^V\} + (1-q(\theta))J^V\Big].$$

We assume that workers and vacancies bargain on the wage to maximize a Nash product. A free entry condition is imposed so that rents are exhausted, as in the main content of our paper. The definition of a steady-state equilibrium can be found in Definition 1 in the main content of the paper. The steady-state equilibrium is characterized by the following equations:

$$\frac{\delta^{R1-\varepsilon}}{1-\varepsilon} - \frac{b^{1-\varepsilon}}{1-\varepsilon} = \frac{\theta\eta}{1-\eta}c \tag{5}$$

and

$$(r+\lambda)c = q(\theta)\int_{\delta^R}^{\infty}(x - w(x))dF(x), \tag{6}$$

where the wage function $w(\delta)$ implicitly solves the following equation:

$$\frac{w(\delta)^{1-\varepsilon}}{1-\varepsilon} - \frac{\delta^{R1-\varepsilon}}{1-\varepsilon} = \frac{\eta}{1-\eta}(\delta - w(\delta)) \tag{7}$$

for any $\delta \geq \delta^R$. The locus (5) equates the outside option value of workers to the reservation productivity level $\delta^R$. Vacancies make zero profits along the locus (6). The wage equation (7) maximizes the Nash product when unemployed workers and unfilled vacancies meet and bargain on the wage.

We compare the economic consequence of stronger workers' ambiguity aversion and risk aversion. According to Proposition 2 in the main content of the paper, if workers become more ambiguity-averse, (i) the reservation productivity threshold $\delta^{R*}$ decreases, (ii) market tightness $\theta^*$ increases, (iii) wages decreases for all $\delta \geq \delta^{R*}$, and (iv) unemployment $u^*$ decreases. Next, we explore the impacts of an increasing degree of risk aversion on the labor market. Our model is solved numerically. First, we solve the wage equation (7) using the Taylor approximation around the point $\delta = \delta^R$ up to the 10th order. The approximated wage equation is then plugged into the two loci (5) and (6) to solve $\{\delta^R, \theta\}$ simultaneously. The equilibrium objects $\{\delta^{R*}, w(\delta), \theta^*, u^*\}$ are plotted in Figure 1.

Figure 1 shows that $\delta^{R*}$ falls with the degree of risk aversion. Notice that unemployed workers receive unemployment benefit $b$, which is less than the wages employed workers receive. A higher

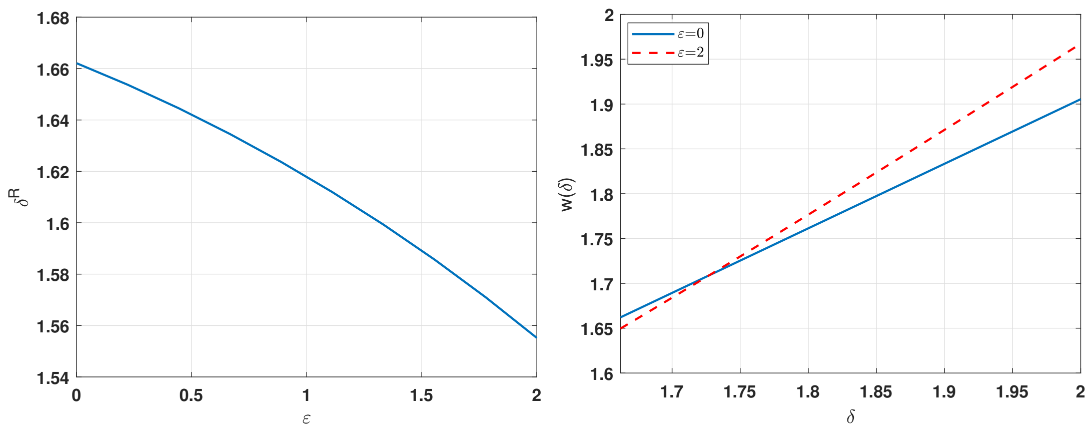


Figure 1: The left panel shows the equilibrium reservation productivity threshold against the degree of risk aversion. The right panel shows the equilibrium wage schedule.

degree of risk aversion discounts a higher income more; as a result, as the degree of risk aversion increases, the expected search benefit becomes relatively less attractive to an unemployment benefit receiver (i.e., the unemployed). From the perspective of workers, the outside option value drops. They are willing to accept a contract with a lower match-specific productivity level. In other words, $\delta^{R*}$ falls with $\varepsilon$. This implication is observationally equivalent to the one under stronger ambiguity aversion.

Next, this figure also indicates that wages under stronger risk aversion are larger if productivity levels are sufficiently high. Due to Nash bargaining, workers are compensated for their outside option value $\delta^R$ and share a fraction of production value $\delta$. Meanwhile, filled vacancies share a fraction of an employed worker's utility value $w(\delta)^{1-\varepsilon}/(1-\varepsilon)$. When the production value is low, the bargained wage largely compensates the outside option value of workers. As discussed above, $\delta^R$ falls with $\varepsilon$; therefore, wages are lower under stronger risk aversion when the production value is low.

Given $\delta$, a worker with stronger risk aversion shares the same amount of production value but shares less utility value with vacancies. As a result, the wage increment is larger for each unit increase in production value. This can be seen by differentiating the wage equation (7) with respect to $\delta$ as follows:

$$\frac{\partial w(\delta)}{\partial \delta} = \frac{\eta}{(1-\eta)w(\delta)^{-\varepsilon} + \eta} > 0.$$

This wage increment rises with the degree of risk aversion in the wage equation. In contrast, $w(\delta) = (1-\eta)\delta^R + \eta\delta$ under ambiguity aversion, in which the parameter governing the degree of ambiguity aversion has no direct impact on the wage increment. While more risk-averse workers with high productivity levels receive higher wages, wages are lower with stronger ambiguity aversion, regardless of production value. Such an implication is different between the two uncertain preferences.

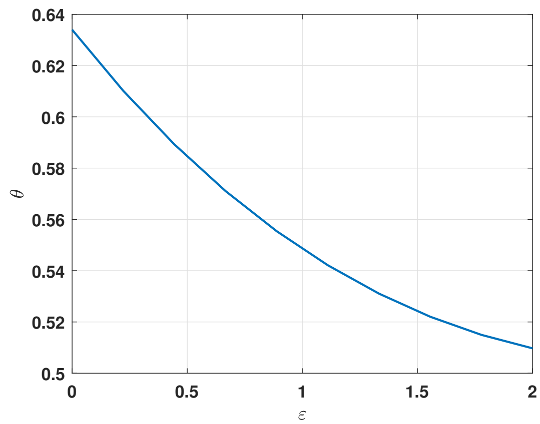


Figure 2: This figure shows the equilibrium market tightness against the degree of risk aversion.

The higher wages arising from stronger workers' risk aversion result in lower market tightness as shown in Figure 2. As explained, wages are higher if production levels are high enough, as the degree of risk aversion increases. Facing a higher expected wage, vacancies have lower expected profits, depressing their supply and thus lowering the market tightness.

The impacts of the two uncertainty preferences on unemployment are analytically different. A steady-state unemployment rate increases with reservation productivity $\delta^R$ and decreases with market tightness $\theta$. We show in Proposition 2 that stronger workers' ambiguity aversion reduces $\delta^{R*}$ and raises $\theta^*$, causing unemployment to decline. Unlike ambiguity preferences, a stronger risk aversion reduces both $\delta^{R*}$ and $\theta^*$, making the impact of risk aversion on unemployment analytically indeterminate. In sum, this appendix shows that not all the impacts of risk and ambiguity preferences on labor market variables, including market tightness and a wage, are observationally equivalent.

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