

# SEARCH RELATIVITY

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## Abstract

This paper proposes a unified theory that explains three seemingly unrelated features of unemployment. First, the higher the educational level, the lower is the unemployment rate by educational attainment (EUR). Any economy with a higher proportion of high-educated workers is expected to have a lower overall unemployment rate (OUR). Nevertheless, the OUR and the proportion of the high-educated are uncorrelated, leading to a statistical puzzle. Second, while the OUR is a weighted average of all the EURs, the data illustrate that there exists a close relationship between the OUR and each EUR beyond an accounting identity. Third, the EUR of the postgraduates is about half the OUR. Our theory explains the statistical puzzle, the fundamental relationship between the OUR and each EUR, and the magic number of the “one-half”. In our model, unemployed workers with heterogeneous human capital levels compete with one another during a job search process. The job-finding rate increases with the relative position in the distribution of search intensity, not its level. The increase in the share of the high-educated improves the relative position of high-educated unemployed workers; meanwhile, it creates a negative externality of the same size on the medium- and the low-educated unemployed, leaving the average job-finding rate and thus the OUR unchanged. Our model derives a novel formula to disaggregate the OUR into various EURs. The null hypotheses that the actual and the predicted EURs are from the same distribution cannot be rejected for high-school graduates and bachelor’s degree holders in 46 and 43 out of 50 states.

**Keywords:** Relative Search Intensity; Unemployment Distribution.

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# 1 Introduction

Back to the days of [Marx \(1923\)](#) and [Keynes \(1937\)](#), economists were concerned with wages and unemployment rates. Since then, these two variables have been at the heart of economic literature. For the theoretical convenience, plenty of earlier works study an average wage level and an unemployment rate of an economy, abstracting the discussion on their distributions. Since the past decades, wage distributions have received lots of attention from both theoretical works ([Galor and Zeira, 1993](#); [Burdett and Mortensen, 1998](#); [Krusell and Smith, 1998](#); [Postel-Vinay and Robin, 2002](#); [Moscarini and Postel-Vinay, 2013](#)) and empirical studies ([Juhn et al., 1993](#); [DiNardo et al., 1996](#); [Katz et al., 1999](#); [Lemieux, 2006](#); [Autor et al., 2008](#)). Nevertheless, works, especially theoretical ones, that study an unemployment distribution across various demographic groups and its properties are surprisingly sparse.<sup>1</sup>

## 1.1 Three Seemingly Unrelated Features of Unemployment

In particular, this paper studies the relationship between the unemployment rate by educational attainment (EUR) and the overall unemployment rate (OUR). We motivate this study by documenting three sets of observations in the United States: (i) the statistical puzzle of an unemployment accounting identity, (ii) the fundamental relationship between the OUR and each EUR, and (iii) the magic number of “one-half”.<sup>2</sup>

**The Statistical Puzzle of an Unemployment Accounting Identity.** First, Figure 1 shows that the EUR decreases with educational level. This negative relationship holds for more than twenty consecutive years.<sup>3</sup> Given an unemployment accounting identity, any economy with a higher proportion of high-educated workers is expected to have a lower OUR.<sup>4</sup> Surprisingly, Figure 2 illustrates that the OUR and the distribution of high-educated are uncorrelated. So, the observations lead to two questions: *what economic mechanism makes the EUR decrease with educational level and, simultaneously, the OUR and the distribution of educational attainment uncorrelated?*

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<sup>1</sup>To the best of our knowledge, this paper is the first theoretical paper that primarily investigates the unemployment distribution across educational groups.

<sup>2</sup>More rigorous empirical examinations on the properties of the EUR follows in Online Appendix A.

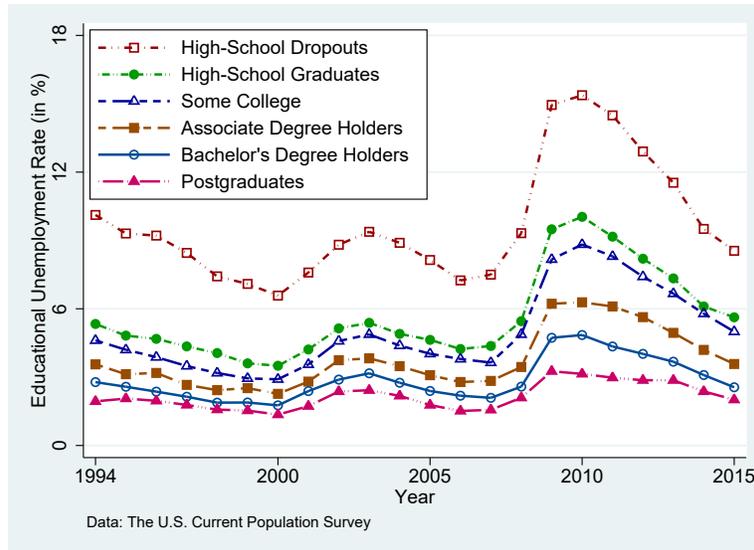
<sup>3</sup>This relationship is also documented in the literature. For example, [Ashenfelter and Ham \(1979\)](#) document that the EUR and the educational attainment are inversely related in the United States. Also, [Topel \(1993\)](#) finds that men of lower wages have a higher risk of unemployment. If men with a higher educational level receive a higher average wage, men with a higher educational level have a lower EUR.

<sup>4</sup>The unemployment accounting identity is given by

$$OUR \equiv \sum_j h_j u_j, \text{ where } \sum_j h_j = 1$$

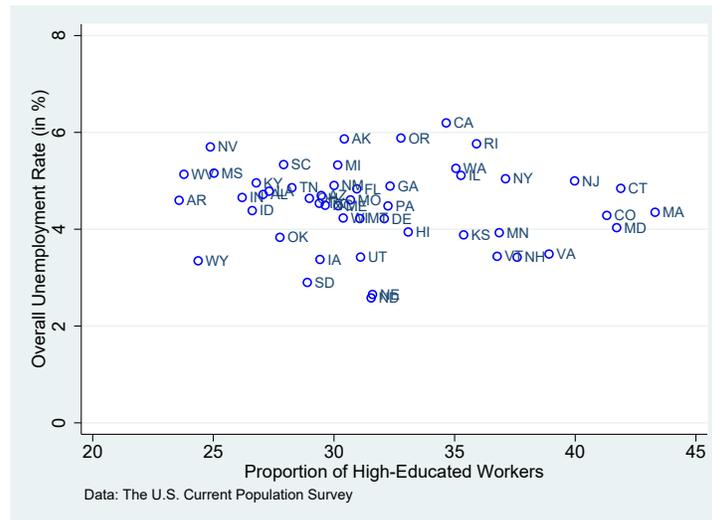
where  $h_j$  and  $u_j$  are the share and the EUR of educational group  $j$ . Undoubtedly, the equation holds by identity.

Figure 1: Yearly EURs in the United States, 1994-2015



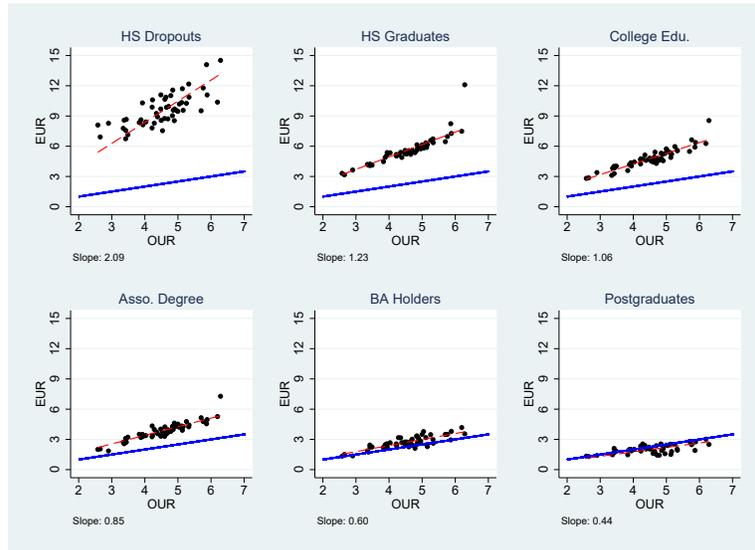
Notes: This figure displays the unemployment rate by educational attainment. Data are collected from the Current Population Survey 1994-2015. Samples are restricted to labor force participants aged 25-60.

Figure 2: The Association between the OUR & the Proportion of High-Educated Workers in the United States, 1994-2015



Notes: Data are collected from the Current Population Survey 1994-2015. Samples are restricted to labor force participants aged 25-60. Each dot illustrates the average OUR and the average fraction of high-educated in each state during 1994-2015. The high-educated are defined as those who are bachelor's degree holders and/or the postgraduates. DC is excluded because it is an outlier.

Figure 3: The Correlation between the OUR and the EURs by State, 1994-2015



Notes: This figure displays the relationship between the overall unemployment rate and the educational unemployment rate by state and educational level. Data are collected from the Current Population Survey 1994-2015. Samples are restricted to labor force participants aged 25-60. The dash line is the fitted value of the EUR and the solid line represents  $EUR = OUR/2$ .

**The Fundamental Relationship Between the OUR and Each EUR.** Figure 3 illustrates the relationship between the OUR and each EUR by state and educational level. The OUR and the EUR of each educational group are positively correlated regardless of educational level and the positive correlation falls with educational level. Strikingly, the OUR and the EUR of each educational group exhibit a close relationship beyond the accounting identity. The observations fit well with one linear relationship between the OUR and each EUR. When regressing EUR on OUR, the derivations of the observations (the dots) from the fitted values (which are linked by the dash line) are surprisingly small, except high-school dropouts. If the EURs are related to the OUR only by the accounting identity, the slopes in Figure 3 might closely reflect the proportion of each group. While the slope decreases with educational level, the proportion is not. Also, some of the slopes exceed a unity and some over two. Therefore, the slopes by no means reflect the proportion of each educational group. Undoubtedly, the accounting identity relates the OUR to the EUR of all the six educational groups, not each of them. This paper answers the underlying questions: *why are the OUR and each EUR positively correlated, why are the positive correlation less pronounced for a higher educational group, and, more importantly, what mechanism governs the fundamental relationship between the OUR and each EUR?*

**The Magic Number of “One-Half”.** Figure 1 suggests that the higher the educational

level, the lower is the EUR. While the postgraduates are the highest educational level in all states, their EUR is supposed to be the lowest. But what determines the level of the lowest EUR in any economy? In Figure 3, the solid line represents the mathematical relationship in which the EUR is half the OUR. Clearly, points lie around this solid line for the postgraduates, suggesting that the EUR of the postgraduates is about half the OUR. More interestingly, this “one-half” is commonly seen not only at the country level (the United States as a whole), but also at the state level (50 different local labor markets). This paper answers the questions: *why is the lowest EUR half the OUR, can it be other numbers, and, fundamentally, what mechanism creates the magic number of this “one-half”?*

While the three sets of observations seem irrelevant to one another, it is unsurprised that the extant literature is silent on any one set of these features. This paper proposes a unified theory to explain all these features, the *three seemingly unrelated* features of unemployment. Despite a voluminous literature on the OUR (Ljungqvist and Sargent, 2008; Elsby et al., 2009; Davis et al., 2010; Shimer, 2012; Sahin et al., 2014), the discussion on the EURs is rare in the literature. The OUR is indeed a weighted average of all EURs and the weight is the fraction of each educational group in the population, which is accessible and rather stable along its trend. Once the properties of the EURs are well understood, the properties of the OUR would be mastered well, not vice versa. Therefore, to understand these *three seemingly unrelated* features of unemployment not only opens the “black box” of the EURs but it also complements the literature on the OUR and enhances our understanding of the functioning of the labor market.

## 1.2 Search Relativity Theory

To answer a series of the research questions, we construct a simple Diamond-Mortensen-Pissarides model in explaining the features of unemployment. The key innovation of this paper is that a matching technology and a relative position of one’s search intensity in an economy jointly determine a job-finding rate. The former factor is standard in the search and matching literature (Rogerson et al., 2005; Rogerson and Shimer, 2011). The latter feature is the main departure of this paper from the literature. An unemployed worker submits his own search intensity to maximize his value function. A higher rank of search intensity will lead to a higher job-finding rate; therefore, this job-finding rate depends largely on others’ strategy. We can interpret the job seeking game as an auction for a higher job-finding rate: an unemployed worker (a bidder) with the highest bid of intensity is rewarded with the highest rate, a bidder with the second highest bid is rewarded with the second highest rate, and so on. In pursuit of a higher job-finding rate, the unemployed needs to have a slightly better curriculum vitae, perform slightly better in a job interview, and devote slightly more time to seek jobs than the candidates ranked slightly higher in the intensity ladder. To

increase one's search intensity without climbing up the intensity ladder, therefore, does not enhance one's job-finding rate. The determination of the optimal search intensity thus requires unemployed workers to consider both the marginal search cost and whether an additional search effort allows them to climb up the intensity ladder.

According to our theory, the EUR of a higher educational group is lower because of their higher relative position in the intensity ladder (hereafter, we call it search relativity if appropriate), not because of their higher search intensity level. Workers with a higher educational group tend to have a higher productivity level and thus wage; therefore, the higher search benefit incents them to search more intensively. This higher search intensity rewards them a higher job-finding rate because of their higher relative position in the search intensity distribution.

Although workers with a higher educational level search more intensively, a higher fraction of high-educated workers does not imply a lower OUR. The increase in one's search intensity does enhance his search relativity. Meanwhile, it creates a negative externality of the same size on others: there exist workers whose ranking declines. Consequently, the two forces cancel and the OUR is independent of the fraction of high-educated workers.

The magic number of one-half is attributed to the striking result, in which the job-finding rate of the postgraduates is always twice the average rate of the economy as a whole. Recall that the job-finding rate is proportional to the relative position of the search intensity distribution in the unemployment. Since the postgraduates possess the highest search benefit, they tend to search the most intensively and thus rank top in the search intensity ladder. Therefore, the cumulative distribution function of the search intensity for the postgraduate is always one. We also show that the expectation of any cumulative distribution function always equals one-half, regardless of the distribution of educational attainment. Consequently, the postgraduates always get a job twice as fast as the average of the economy, explaining why the lowest EUR is about half the OUR.

### 1.3 Implications

Our theory departs from the conventional wisdom on the determinants of matching functions. Despite no economy-wide experiment, economists and non-economists observe that those who search more intensively tend to get rid of unemployment faster. This might formulate our belief that the level of search intensity determines one's job-finding rate. Our theory, on the contrary, proposes that the difference in a job-finding rate is (in large part) attributed to search relativity, not the level of search intensity. For example, [Kroft and Pope \(2014\)](#) find that an increasing popularity of a website like Craigslist largely reduces search cost in a job search process but has no impact on the unemployment rate. This contradicts to the conventional thought: the reduction in search cost is expected to cause the search

intensity level to rise, thereby lowering the OUR. Nevertheless, Kroft and Pope (2014) find that an increasing popularity of a job-finding website does not reduce the OUR. Our proposed theory provides one of the potential explanations to this so-called puzzle: the OUR is uncorrelated with the average level of search intensity. Theoretically, the reduction in search cost might increase the level of search intensity of the Craigslist's user. The increase in their search intensity levels does enhance their relative positions in the search intensity ladder. Meanwhile, it creates a negative externality of the same size on the nonuser of Craigslist: there exist workers whose ranking declines. Overall, a rise in the job-finding rate for a certain group of the unemployed has no effect on the OUR.

Our model is not only theoretically appealing, but it also explains the three sets of observations and leads to several striking results. First, this model derives a novel formula that relates each of the EURs to the OUR except high-school dropouts. We evaluate the derived EURs using the US data during 1994–2015. For each educational group in each state, most the null hypotheses, in which the actual and the derived EURs are from the same distribution, cannot be rejected at any conventional significance level. For example, the null hypotheses cannot be rejected for high-school graduates and bachelor's degree holders in 46 and 43 out of 50 states at five percent significance level. This implies that our model does well predict not only the trends in the EURs, but also their magnitudes over the past two decades. Moreover, the formula requires only two input variables: the OUR and the distribution of educational level, both of which are easily accessible. Using the least number (two) of input variables, our simple formula allows economists, policymakers, and the public to accurately map the OUR to a more relevant information to the public — the EUR. Furthermore, we derive another approximation formula of the EUR of the postgraduates. In particular, their EUR approximately equals  $\frac{u}{2-u}$ , where  $u$  is the OUR. Again, the derived formula, though simple, is so accurate that the statistical tests cannot distinguish the distribution between the actual and the predicted values. The striking results suggest that the search relativity is the economic mechanism that governs the fundamental relationship between the OUR and the EUR, answering the questions stemming from the second set of observations.<sup>5</sup>

Second, this model derives the formula of the “fundamental frictional unemployment rate”, which is defined as the unemployment rate at which the associated unemployment spell cannot be further shortened simply by increasing search effort or the level of human capital of workers. In other words, this paper computes the unemployment rate that is driven sheerly by a search friction (a matching technology), abstracting other factors like search effort or ability. While the difference in the OUR across countries could be attributed to the difference in search friction, workers' search intensity, and their human capital, it is

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<sup>5</sup>Here, this paper by no means implies the search relativity is the only factor in determining the job-finding rate.

difficult to learn about the search friction simply by comparing the OURs across countries. Similarly, whether a rise in the OUR in recession results from the increase in search friction or the decline in the incentive to seek job remains an empirical question and is difficult to investigate. The fundamental frictional unemployment rate allows economists to quantify the frictional unemployment rate so as to study the properties of search friction in the labor market. Furthermore, this measure allows us to disentangle the overall unemployment that is attributable to other factors such as rationing unemployment (Michaillat, 2012), mismatch unemployment (Sahin et al., 2014), and ambiguous unemployment (Chan and Yip, 2017) from frictional unemployment.

## 1.4 Related Literature

To the best of our knowledge, there exists no work that attempts to explain any one set of the features of unemployment documented in this paper. Because these features are seemingly irrelevant to one another, it is unsurprised that no empirical studies bring these pieces of features into one scientific work and discuss. With no specific motivation in explaining the unemployment distribution, it is also unsurprised that no theoretical model could explain any one set of the features. For example, Acemoglu (2001), Albrecht and Vroman (2002), Wong (2003), and Dolado et al. (2009) are works with a finite number of worker types in the random search literature. With a special interest in the wage distribution, most of these works perform well in modeling between and within educational group wage inequality. While an unemployment distribution is out of the scope, most of these works unsurprisingly predict that the unemployment rates for different types of workers are identical.

In the directed search literature, recent works like Eeckhout and Kircher (2010) and Peters (2010) model an economy with a continuum of worker types.<sup>6</sup> In Peters (2010), a continuum of workers of different types directs their applications to different jobs. Firms value workers' characteristics differently and give a job offer to the job seeker they most value. His model, unlike the other directed search literature, succeeds in getting rid of the tradeoff between higher wages and lower unemployment rates, resolving the common problem in this literature. As pointed out by the author, the setting that firms could not condition wages on workers' characteristics is awkward and might slightly depart from the reality.<sup>7</sup> With no specific interest in the EUR, it is unsurprised that the features of EURs are rarely discussed in this literature.

Our theory is closely related to the literature on empirical matching functions. While it is an empirical challenge in observing and measuring the level of search intensity, this literature either ignores search intensity or utilizes a proxy for the intensity level (Petrongolo

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<sup>6</sup>Eeckhout and Kircher (2010) also allows for a continuum of firm types.

<sup>7</sup>The model could predict that wages and skill levels are positively related as the high-skilled unemployed tend to direct their search to the positions of higher wages.

and Pissarides, 2001; Shimer, 2004; Bloemen, 2005; Stevens, 2007; Davis et al., 2012). The success of our theory in explaining the three sets of observations of the unemployment suggests that search relativity, not its level, is one of the key input variables in matching functions.

The basic model environment is described in Section 2, followed by the construction of a steady state Nash equilibrium in Section 3. Our model is shown to capture the features of unemployment in Section 3. Beyond the trends, Section 4 evaluates the magnitude of the unemployment distribution and the EURs derived by the model. The derived formulas are shown to perform well (if not considered as nearly perfectly) using the United States Current Population Survey 1994-2015. In addition, the section derives the formula of the Fundamental Frictional Unemployment Rate. Section 5 concludes the paper.

While Online Appendix A illustrates further evidence on the first two sets of the features, Online Appendix B provides examples to show that the existing search and matching models fail to capture the features. Online Appendix C presents figures in supporting the argument in Online Appendix A. Online Appendix D, E, and F present figures that evaluate the predictability of unemployment distribution, a disaggregation theory, and a disaggregation approximation theory, respectively, all of which are used in Section 4.

## 2 The Basic Model

The proposed theory is presented through the workhorse of a search equilibrium model. We aim to construct a simple model to shed light on the mechanism through which the search relativity affects job-seeking behaviors. Despite the abstraction of other labor market features, our model is flexible, could be easily extended to incorporate those other features, and, of paramount importance, captures well the relation between the OUR and each EUR. This section aims to present the model, in which the job-finding rate increases with the search relativity, and the technical details required to understand the search relativity theory.<sup>8</sup>

Consider an economy with a continuum of utility maximizing workers and a continuum of profit-maximizing firms. Without loss of generality, the mass of workers is fixed and normalized to unity. Workers and firms are risk neutral and share an identical real interest

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<sup>8</sup>In Online Appendix B, we demonstrate that if the job-finding rate increases with the search intensity level, the search equilibrium models fail to explain the features of unemployment.

rate  $r$ .<sup>9</sup> Workers are either employed or unemployed.<sup>10</sup> Workers are employed if they hold a position in a firm, and unemployed otherwise. A firm could be filled by at most one worker, and any worker can take up at most one position. Since one firm is basically equivalent to one vacancy, we follow the tradition in this literature to address a firm as a vacancy to avoid confusion. Hence, vacancies are either filled or unfilled.

Workers differ in their human capital level  $\delta$ . Denote  $H(\delta)$  as the cumulative distribution function of  $\delta$ , and the corresponding density function as  $h(\delta)$ . We assume that the human capital level is finite; otherwise, its likelihood is zero. Hence, the assumption on the distribution function of  $\delta$  is  $\lim_{\delta \rightarrow \infty} \delta h(\delta) = 0$ .<sup>11</sup> To simplify the analysis, we assume that the lower support of  $H(\delta)$  exceeds unemployment benefits  $z$  so that workers are willing to sign a contract during a job interview.<sup>12</sup>

Denote  $J^E(\delta)$  and  $J^U(\delta)$  as value functions of employment and unemployment respectively. An employed worker with  $\delta$  receives a wage  $w$  and faces a separation shock at a Poisson rate  $\lambda$ . When the shock arrives, the employed worker becomes unemployed. Hence, the Hamilton-Jacobian-Bellman equation can be written as follows.

$$rJ^E(\delta) = w(\delta) + \lambda(J^U(\delta) - J^E(\delta)) \quad (1)$$

An unemployed worker receives an unemployment benefit  $z \in \mathbb{R}_+$ . He pays a search cost  $C : \mathbb{R}_+ \mapsto \mathbb{R}_+$  and selects the optimal level of search intensity  $s$ . Assume that the search cost is zero if  $s = 0$  and the function is strictly convex. Hence,  $C(0) = 0$ ,  $C'(s) > 0$

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<sup>9</sup>This simplification is common in this literature (Mortensen and Pissarides, 1994; Moen, 1997; Moscarini, 2005; Rogerson et al., 2005; Gonzalez and Shi, 2010; Fujita and Ramey, 2012; Michaillat, 2012). Applications of the search and matching model also assume agents to be risk-averse such as the literature that investigates the optimal unemployment benefits with search frictions (Fredriksson and Holmlund, 2006; Guerrieri et al., 2010). Recent literature also investigates job search behaviors with the preference of ambiguity aversion (Chan and Yip, 2017). Our model can be easily extended to incorporate agents' ambiguity aversion; we do not do so because the modification of agents' preference towards ambiguity complicates our model without providing a richer economic intuition in our context.

<sup>10</sup>The model assumes that no decision on labor supply, either the number of working hours or labor force participation, is made. This simplification is standard (Mortensen and Pissarides, 1994; Shimer, 2005; Hall, 2005; Hagedorn and Manovskii, 2008; Hall and Milgrom, 2008; Fujita and Ramey, 2012; Michaillat, 2012), and is in line with empirical regularities: cyclical variations in total working hours (unemployment) basically arise from changes in the number of employment but not changes in working hours per worker (labor force participation) (Shimer, 2010).

<sup>11</sup>The assumption is a sufficient condition for the existence of the equilibrium in a steady state, which will be shown later. In fact, it is natural to assume the distribution to be either a log-normal distribution or a Pareto distribution, both of which satisfy the assumption on  $H(\delta)$ . Indeed, a normal distribution and an exponential distribution also satisfy this assumption. We do not restrict the distribution of  $\delta$  for a broader set of readers who are interested in the steady state equilibrium of a similar framework with other distributions.

<sup>12</sup>This assumption rules out the possibility of not participating in a labor market. As mentioned, the decision on labor force participation is beyond the scope of this paper. Our model matches well with the data under this simplification probably because cyclical variations in unemployment basically arise from changes in the number of employment but not changes in labor force participation (Shimer, 2010).

and  $C''(s) > 0$  for all  $s \in \mathbb{R}_+$ . An unemployed worker transits from unemployment to employment at a rate of  $F(s)p$ , where  $p \in (0, 1)$  is a matching rate of the economy.<sup>13</sup> In contrast to the existing literature, the job-finding rate also depends on  $F(s)$ , which describes the relative position of search intensity in unemployment. To capture the properties of a job-finding rate, we assume that  $F(s)$  is a cumulative distribution function of search intensity in unemployment for several reasons. First,  $F(s)$  increases in  $s$  so that the job-finding rate increases with the rank in the search intensity ladder. Second, the lower bound of the  $F(s)$  is zero so that the job-finding rate is zero if no search effort is made in a job search process. Third, its upper bound is one to guarantee that a job-finding rate  $F(s)p$  less than one. The job-finding rate is less than one even if  $F(s) = 1$  due to another component  $p$  of the search function in the matching function that is common to the entire economy.<sup>14</sup> It is noteworthy that the distribution of search intensity  $F(s)$  is endogenized. The selection of the optimal search intensity largely depends on the relative position in the ladder. Similar to an auction, each unemployed worker submits his own search intensity to bid for a higher job-finding rate  $F(s)p$ , and is rewarded with the rate  $F(s)p$ . In the Nash steady state equilibrium, each unemployed worker chooses  $s$  to maximize the value of unemployment given the optimal search intensity  $s^*$  of other unemployed workers.

Given  $J^E(\delta)$  and others' best response function  $s^*$ , an unemployed worker of type  $\delta$  chooses his action  $s$  to maximize his value of unemployment as follows.

$$rJ^U(\delta) = \max_s \{z - C(s) + F(s)p(J^E(\delta) - J^U(\delta))\} \quad (2)$$

Regarding vacancies, we assume that their numbers are fixed. When a vacancy is filled by a worker of type  $\delta$ , it generates a production value  $\delta$  and pays a wage  $w$  to the worker.<sup>15</sup> A filled vacancy faces a separation shock at a rate of  $\lambda$ . When the shock arrives, a filled vacancy becomes unfilled, and receives zero profits. Hence, an asset value of a filled vacancy is written as follows.

$$rJ^F(\delta) = \delta - w(\delta) - \lambda J^F(\delta) \quad (3)$$

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<sup>13</sup>Since endogenizing market tightness does not provide additional economic insight of this paper but significantly complicates the analysis, we assume that the job-finding rate does not depend on market tightness. In fact, endogenizing market tightness does not alter the disaggregation theory.

<sup>14</sup>One might be concerned that workers of different human capital levels might not compete with one another in a job search process. In reality, jobs are not perfectly segregated by human capital level. Therefore, it is reasonable that the high-school dropouts and the high-school graduates might compete for a similar job type, and the bachelor's degree holders and the postgraduates might also perform a similar task in their positions. Meanwhile, it is rarely to see that the high-school dropouts and the postgraduates compete for the same job. We will show that the steady state Nash equilibrium is in line with these observations. To decide whether to increase one's search intensity, a worker has to consider whether such increase improves his ranking in the ladder. Also, we will show that workers with similar human capital level are in a similar rank; therefore, workers with significant difference in human capital will not compete with one another in the equilibrium, in line with the reality.

<sup>15</sup>In fact, the implication of this model does not change if the production value is equal to  $f(\delta)$ , where  $f'(\delta) > 0$ .

Following the existing literature, we assume that wages are determined by maximizing the generalized Nash product. Consequently, expected gains from search are split according to the generalized Nash bargaining solution as follows.

$$J^E(\delta) - J^U(\delta) = \beta (J^E(\delta) - J^U(\delta) + J^F(\delta)) \quad (4)$$

where  $\beta \in (0, 1)$  is a bargaining power of workers. The higher the value of  $\beta$ , the greater is the workers' bargaining power. Equating flow in and flow out of unemployment, a steady state unemployment rate is given by

$$\lambda(1 - u) = \underbrace{\int F(s^*(\delta))pdG(\delta)}_{\text{Average job-finding Rate}} \times u \quad (5)$$

where  $G(\delta)$  is a cumulative distribution function of the unemployed workers of type  $\delta$ , with  $g(\delta)$  the corresponding probability density function. Hence,  $G(\delta)$  measures the unemployment distribution of human capital. The LHS and the RHS of the equation (5) is the flow in and flow out of unemployment, where  $p \int F(s^*(\delta))dG(\delta)$  is the average job-finding rate, and  $s^*(\delta)$  is the optimal search intensity submitted by the unemployed worker of type  $\delta$ . Similarly, a steady state EUR is given by

$$\lambda(h(\delta) - g(\delta)u) = F(s^*(\delta))pg(\delta)u \quad (6)$$

The LHS captures the number of employed workers of type  $\delta$  flowing into unemployment, where  $h(\delta) - g(\delta)u$  is the measure of employed workers of type  $\delta$ .  $F(s^*(\delta))p$  is a job-finding rate of the unemployed workers of type  $\delta$ . So, the RHS captures their flow out of unemployment, where  $g(\delta)u$  is the measure of unemployed workers of type  $\delta$ .

### 3 Characterization of the Steady-State Nash Equilibrium

This section purposes to characterize the steady state Nash equilibrium of our model, explore the mathematical properties of the steady state Nash equilibrium, and provide economic intuitions as to why the search relativity could explain the *three seemingly unrelated* feature of unemployment. We begin by defining the steady state Nash equilibrium.

**Definition 1.** A steady state Nash equilibrium is defined as  $\{s(\delta), J^E(\delta), J^U(\delta), J^F(\delta), u, g(\delta)\}$ , for all  $\delta \geq z$ ,

1. (Optimal Search Intensity):  $s(\delta)$  maximizes  $J^U(\delta)$  given  $s^*(\delta)$  of other unemployed

workers;

2. (Value Functions):  $J^E(\delta)$ ,  $J^U(\delta)$ , and  $J^F(\delta)$  satisfy equations (1)-(3);
3. (Rent-Sharing):  $w(\delta)$  maximizes the generalized Nash product, satisfying the sharing rule (4);
4. (Steady-State Accounting):  $u$  and  $g(\delta)$  satisfy equations (5) and (6).

Using equations (1), (3), and (4), the wage equation can be written by

$$w(\delta) = rJ^U(\delta) + \beta(\delta - rJ^U(\delta)) \quad (7)$$

A wage is equal to an outside option value plus a fraction of economic rent. Using equation (2) and (7), the outside option value is given by

$$rJ^U(\delta) = \max_s \left\{ z - C(s) + F(s) \frac{\beta p}{r + \lambda} (\delta - rJ^U(\delta)) \right\} \quad (8)$$

**Lemma 1.** *If  $\delta_1 > \delta_2$ ,  $s^*(\delta_1) \geq s^*(\delta_2)$  in a steady state Nash equilibrium.*

*Proof.* Denote  $\Phi(\delta)$  as  $\frac{\beta p}{r + \lambda} (\delta - rJ^U(\delta))$ . If  $\partial\Phi(\delta)/\partial\delta \leq 0$ , then  $\partial rJ^U(\delta)/\partial\delta \geq 1$ . Applying the envelope theorem to equation (8),  $\partial\Phi(\delta)/\partial\delta \leq 0$  implies that  $\partial rJ^U(\delta)/\partial\delta \leq 0$ . A contradiction results. Hence, we can conclude that  $\partial\Phi(\delta)/\partial\delta > 0$ . We now prove that  $s^*(\delta_1) \geq s^*(\delta_2)$  by contradiction. Suppose it is not the case. Given  $\delta_1 > \delta_2$ , there exists a steady state Nash equilibrium such that  $s_1 < s_2$ , where  $s_i$  is denoted as the optimal search intensity of the worker of type  $\delta_i$ . Workers of type  $\delta_2$  picks  $s_2$  because  $F(s_2)\Phi(\delta_2) - C(s_2) \geq F(s_1)\Phi(\delta_2) - C(s_1)$ . Therefore, we have

$$\begin{aligned} (F(s_2) - F(s_1))\Phi(\delta_2) &\geq C(s_2) - C(s_1) \\ (F(s_2) - F(s_1))\Phi(\delta_1) &> C(s_2) - C(s_1) \\ F(s_2)\Phi(\delta_1) - C(s_2) &> F(s_1)\Phi(\delta_1) - C(s_1) \end{aligned}$$

The second inequality arises because  $\Phi(\delta)$  is strictly increasing in  $\delta$  and  $\delta_1 > \delta_2$ . According to the last inequality, it is strictly better off for workers of type  $\delta_1$  to choose  $s_2$  instead of  $s_1$ . Therefore, we can conclude that  $s^*(\delta_1) \geq s^*(\delta_2)$  if  $\delta_1 > \delta_2$  in a steady state Nash equilibrium.  $\square$

Intuitively, the higher the human capital, the higher is the bargaining wage due to rent-sharing. Hence, an unemployed worker with a higher  $\delta$  has a higher net search benefit. Consequently, they are willing to pay a search cost at least as high as those with lower  $\delta$ . Lemma 1 shows that if the net search benefit is sufficiently large for workers of type  $\delta_2$  to search at  $s_2$ , it is also high enough for a worker of type  $\delta > \delta_2$  to search at the same intensity

$s_2$ . We will show that  $ds^*(\delta)/d\delta > 0$  is the rule to construct the strategy in a steady state Nash equilibrium. Furthermore, Lemma 1 implies that all the unemployed workers of type  $x \leq \delta$  will search with  $s^*(x)$  not higher than  $s^*(\delta)$ , meaning that  $G(\delta) = F(s^*(\delta))$ . Hence, in the equilibrium, the distribution of the search intensity coincides with the distribution of human capital in the unemployment.

Differentiating both sides of  $G(\delta) = F(s^*(\delta))$  with respect to  $\delta$  yields  $g(\delta) = f(s^*(\delta))ds^*(\delta)/d\delta$ . Using  $F(s^*(\delta)) = G(\delta)$ ,  $G(z) = 0$  and equation (5), a steady state unemployment rate is given by

$$u = \frac{\lambda}{\lambda + \int_z^\infty F(s^*(x))pdG(x)} = \frac{\lambda}{\lambda + p \int_z^\infty G(x)dG(x)} = \frac{\lambda}{\lambda + \frac{1}{2}p} \quad (9)$$

Ostensibly, the OUR (9) is parallel to the one derived from the search and matching literature:  $u$  strictly increases with  $\lambda$  but decreases with  $p$ . Using  $F(s^*(\delta)) = G(\delta)$ , the EUR (6), and the OUR (9), Appendix 6.1 gives the proof that  $G(\delta)$  can be written as follows.

$$G(\delta) = \frac{u}{2(1-u)} \left( \sqrt{1 + \frac{4(1-u)H(\delta)}{u^2}} - 1 \right) \quad (10)$$

Differentiating  $G(\delta)$  with respect to  $\delta$  and rearranging terms, the EUR  $u_\delta$  is given by

$$u_\delta \equiv \frac{g(\delta)u}{h(\delta)} = \left( 1 + \frac{4(1-u)H(\delta)}{u^2} \right)^{-\frac{1}{2}} \quad (11)$$

which is a function of the OUR and the distribution of human capital. Thus far, the steady state OUR (9) and EUR (11) have been derived.

Differentiating equation (2) with respect to  $s$  and using equations (1) and (7) give the best response function of workers of  $\delta$  as follows.

$$\underbrace{C'(s^*(\delta))}_{\text{Marginal Search Cost}} = f(s^*(\delta)) \frac{\beta p}{r + \lambda} (\delta - rJ^U(\delta)) \underbrace{\text{Marginal Search Benefit}} \quad (12)$$

The optimal search intensity equates a marginal search cost to a marginal search benefit. Using  $f(s^*(\delta))ds^*(\delta)/d\delta = g(\delta)$  and equation (12), one could verify that search effort strictly increases with  $\delta$ .

$$\frac{ds^*(\delta)}{d\delta} = \frac{\beta p g(\delta) (\delta - rJ^U(\delta))}{(r + \lambda)C'(s^*(\delta))} > 0 \quad (13)$$

Using  $f(s^*(\delta))ds^*(\delta)/d\delta = g(\delta)$  and equation (2), (10), and (11), the optimal  $s^*(\delta)$  in equation (12) can be found by solving  $C \equiv C(s^*(\delta))$  in the following first order linear

differential equation.<sup>16</sup>

$$\frac{dC}{d\delta} = T(\delta)(\delta - z) + T(\delta)C \quad (14)$$

where  $T(\delta) \equiv \frac{\beta p h(\delta)}{(r+\lambda)u\Phi_1(\delta)(1+\Phi_2(\delta))}$ ,  $\Phi_1(\delta) \equiv \sqrt{1 + \frac{4(1-u)H(\delta)}{u^2}}$ ,  $\Phi_2(\delta) \equiv \frac{\beta\lambda}{r+\lambda}(\Phi_1(\delta) - 1)$ . The initial condition is given by  $C(s^*(z)) = 0$ . The unique solution to this initial value problem is given by

$$C(s^*(\delta)) = \int_z^\delta T(x')(x' - z)e^{\int_{x'}^\infty T(y)dy} dx' \quad (15)$$

Denote  $\Omega(\delta, p)$  as the RHS of the above equation. Appendix 6.3 shows that  $\lim_{p \rightarrow 0} \Omega(\delta, p)$  is zero,  $\lim_{p \rightarrow \infty} \Omega(\delta, p)$  is positive finite, and  $\lim_{p \rightarrow \infty} \frac{\partial \Omega(\delta, p)}{\partial p}$  is zero. The assumption  $\lim_{\delta \rightarrow \infty} \delta h(\delta) = 0$  ensures that the solution is finite for all  $\delta \geq z$ . Since  $C(s^*(\delta))$  strictly increases with  $s^*$ , the unique  $s^*$  is given by

$$s^*(\delta) = C^{-1}(\Omega(\delta, p)) \quad (16)$$

$\partial \Omega(\delta, p) / \partial p \geq 0$  implies that  $s^*(\delta)$  is increasing in  $p$  for all  $\delta \geq z$ . With a higher  $p$ , an increase in a job-finding rate and thus a marginal search benefit from an increase in the relative ranking in intensity is amplified. Unemployed workers would have sufficient incentive to deviate to the new Nash equilibrium with a higher  $s^*(\delta)$ . Interestingly, in response to the rise in  $p$ , all unemployed workers increase their search intensity but the ranking of no one gets improved.

Now, we show that the marginal search cost (MSC) intersects the marginal search benefit (MSB) at the  $s^*(\delta)$  where no worker of type  $\delta$  has any incentive to deviate from this strategy. Suppose that the MSB curve cuts the MSC curve from below. Since  $f(s)$  is bounded above, given  $rJ^U(\delta)$  the marginal search benefit is bounded above. However,  $\lim_{s \rightarrow \infty} C'(s) = \infty$ . Therefore, if there exists an intersection such that the MSB curve cuts the MSC curve from below, there exists another intersection at  $s > s^*(\delta)$ . A contradiction results because the uniqueness of the  $s^*(\delta)$ , given by equation (16), implies that the MSC and the MSB curve intersect once in the interval  $\mathbb{R}_{++}$ . Now, we can conclude that  $s^*(\delta)$  happens at the intersection where the MSC curve cuts the MSB curve from below. Workers of  $\delta$  have no incentive to cut search intensity because the marginal search benefit is greater than the marginal search cost for all  $s \in (0, s^*(\delta))$ . Also, they are reluctant to increase search intensity because the marginal search benefit is less than the marginal search cost for all  $s > s^*(\delta)$ . Given that other workers follow the strategy  $s^*(\delta)$  from equation (16), workers of type  $\delta$  have no incentive to deviate from the search effort  $s^*(\delta)$

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<sup>16</sup>The derivation is given in the Appendix.

that satisfies equation (12) and is given by (16).

**Proposition 1. (Existence of the Equilibrium)** *There exists a steady state Nash equilibrium defined in Definition 1. A steady state OUR and the EUR are given by equation (9) and (11) respectively.*

Proposition 1 states that there exists an equilibrium. Nevertheless, it does not guarantee the uniqueness of the equilibrium. In fact, for all  $\delta \geq z$ ,  $s^*(\delta) = 0$  will yield another steady state Nash equilibrium that satisfies Definition 1. Given that all other unemployed workers do not search, the best response function of workers of any type is  $s^*(\delta) = 0$ , and the corresponding job-finding rate  $F(s)p$  is equal to  $p$ , which is the highest. If a worker deviates from the equilibrium and search at any positive intensity  $s^*(\delta) > 0$ , his job-finding rate remains  $p$ . However, he has to pay a positive search cost  $C(s^*(\delta)) > 0$ . Hence, given that no unemployed workers search at positive intensity, deviating from  $s^*(\delta) = 0$  lowers one's payoff. In this equilibrium,  $s^*(\delta) = 0$  and  $C(s^*(\delta)) = 0$ . The value function of unemployment of type  $\delta$  reduces to

$$rJ^U(\delta) = z + p(J^E(\delta) - J^U(\delta))$$

which is equivalent to the value function of unemployment in a standard search and matching model without search intensity. Our model reduces to the standard model. While this equilibrium is widely discussed in the literature and is shown not to capture any one set of the features of unemployment in Online Appendix B, our attention is paid to its alternative equilibrium, in which the distribution of search intensity is not degenerated. Theorem 1 will summarize the five properties of the derived unemployment rates in the steady state Nash equilibrium.

**Theorem 1. (Properties of Unemployment Rates)** *In a steady state Nash equilibrium,*

1. *the EUR  $u_\delta$  unambiguously decreases with  $\delta$ ;*
2.  *$u$  is independent of the distribution of human capital  $H(\delta)$ ;*
3. *a rise in  $\lambda$  and a fall in  $p$  increase  $u_\delta$  and  $u$ ;*
4. *if  $\frac{2H(\delta)}{u}(\frac{1}{u} - 1) > 1$ , the EUR with a lower  $\delta$  increases more in slumps; and*
5. *the lowest EUR  $u_\delta$  is equal to  $\frac{u}{2-u}$ .*

*Proof.* See the Appendix 6.4. □

**The First Property.** The first statement indicates that the higher the human capital level, the lower will be their unemployment rate. This implication is seemingly no difference from the standard one in the literature in that an unemployed worker with a higher search intensity level is more likely to transit into employment. However, the mechanisms

that drive them to get rid of unemployment faster are different. In the literature, the unemployed worker is more likely to get hired because of the higher level of search intensity. Here, the level of search intensity essentially plays no role in determining one's job-finding rate. Instead, a higher search intensity places an unemployed worker at a higher position in the search ladder, which in turn rewards him a higher job-finding rate. This explains why the higher the educational level, the lower is the EUR.

**The Second Property.** It is noteworthy that  $u$  is a function of a separation rate  $\lambda$  and a matching rate  $p$  that is common to an economy, not the average level of search intensity or the distribution of  $\delta$ . This property is important for three reasons. First, the independence of  $u$  from the human capital distribution explains at least two phenomena. It explains not only why the correlation between the OUR and the distribution of educational attainment is close to zero, as shown in Figure 2, but also why the natural rate of unemployment remains steady for quite a long time (at least 50 years in the U.S.) even though the proportion of high-educated workers increases over time (which in turn increases the average human capital level). Second, one should be aware that it is rather difficult to obtain the information on workers' search intensity and is hard to measure search intensity in practice. The independence guarantees that the derived OUR is free from any function of search intensity, making practitioners easier to uncover the OUR simply with the transition rates, namely the job-finding rate and the separation rate. Third, this independence property is unconditional on the stage of the development of an economy, meaning that the independence in principle holds in economies regardless of the distribution of human capital.

Mathematically, the OUR is negatively associated with the average job-finding rate, given by  $p \int_z^\infty F(s^*(x))dG(x)$ . Since the higher the human capital level, the higher is the search intensity level and thus the relative position in the search intensity distribution. The search intensity distribution coincides with the distribution of human capital in the unemployment. That is,  $F(s^*(\delta)) = G(\delta)$ . The average job-finding rate reduces to  $p \int_z^\infty G(x)dG(x)$ . Since the mean value of any cumulative distribution function is  $1/2$ , the average job-finding rate further reduces to  $p/2$ , independent of the distribution of search intensity or human capital. Intuitively, consider a worker increases his search effort such that his ranking slightly improves. On the one hand, such increase in the search effort shrinks his average unemployment spell. On the other hand, there exist other unemployed workers whose rankings decline in the search effort ladder, lengthening the average unemployment spell of those with the decline in the rankings. More importantly, the negative externality, created by the increase in one's search effort, is of the same magnitude as the improvement in the one's ranking. Overall, the average job-finding rate of an economy is unaffected by the reallocation of the ranking, remaining  $p/2$ . This explains why a rise in one's search effort could shorten his own average unemployment duration but not the unemployment spell of the economy as a whole, leaving the OUR independent of the distribution of human

capital or search effort.

**The Third Property.** The third statement indicates that a higher  $\lambda$  and a lower  $p$  increase the EUR. Intuitively, the higher separation rate increases the flow into unemployment and the lower job-finding rate reduces the flow out of unemployment, both of which lengthen the unemployment duration and increase the EUR. We can also infer that it is the increase in  $\lambda$  and/or the decrease in  $p$  that leads to a rise in the EUR, which in turn increases the OUR in slumps.

**The Fourth Property.** This statement illustrates that the increase in the EUR is more pronounced for workers with a lower human capital value in slumps. Indeed, a fall in  $p$  has two forces on the EUR. First, with a lower  $p$  the marginal effect of the increase in the search relativity  $F(s^*(\delta))$  becomes weaker, closing the gaps in the job-finding rates and thus the EURs in slumps. Second, notice that the difference in the EUR arises from the heterogeneity in the search intensity (which is the first property.) and the heterogeneity happens only in the unemployment, not the employment. If the unemployment rate is higher, there will be more workers experiencing the effect arising from the heterogeneity in the search intensity. Therefore, the reduction in  $p$  will increase in the EUR more for workers with a lower human capital value under a condition. But if  $\frac{2H(\delta)}{u}(\frac{1}{u} - 1) > 1$ , the former effect will be dominated. Therefore, the EUR with a lower  $\delta$  increases more in slumps. The intuition is similar for the rise in  $\lambda$ .

**The Fifth Property.** This property relates the lowest EUR to the OUR. Recall from the first property that the higher the human capital level, the lower is the EUR. Hence, the EUR reaches its minimum when  $\delta$  tends to infinity. Using equation (11), it is straightforward to show that the lowest EUR is  $\frac{u}{2-u}$ , which is close to half the OUR. Notice that the unemployment rate of one group is negatively associated with the average job-finding rate in the group. That is, the higher the transition rate  $F(s^*(\delta))p$ , the smaller is the corresponding the EUR. As shown in Lemma 1, the higher the human capital level, the higher is the search intensity level. Workers with the highest human capital level will search the most intensively in an economy so that they, once get unemployed, rank top in the intensity ladder. Hence, their job-finding rate becomes  $p$ .

Regarding the economy as a whole, the average job-finding rate is equal to  $\int_0^1 F(s^*(\delta))pG(\delta)$ . Again, Lemma 1 implies that the search intensity distribution  $F(s^*(\delta))$  coincides with the human capital distribution in the unemployment  $G(\delta)$ . Therefore, the rate reduces to  $p \int_0^1 G(\delta)dG(\delta)$ , which is equal to  $p/2$ .<sup>17</sup> Consequently, the job-finding rate of the entire population is about half the rate of the workers with the highest human capital level; therefore, the unemployment rate for the workers with the highest human capital about half the OUR. Importantly, this striking result holds regardless of the distribution of human capital.

Clearly, these properties of unemployment in Theorem 1 correspond to the three sets

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<sup>17</sup> $p \int_0^1 G(\delta)dG(\delta) = p \int_0^1 xdx = p/2$ .

of the observations of unemployment. The fourth property is valid only if  $\frac{2H(\delta)}{u}(\frac{1}{u} - 1)$  holds. To match the properties to the observations, the remaining question is how likely the inequality holds in the fourth property. Notice that  $\frac{2H(\delta)}{u}(\frac{1}{u} - 1)$  strictly increases with  $H(\delta)$  and decreases with  $u$ . According to US CPS data, the proportions of high-school graduates or below followed a decreasing trend and reached its lowest point at 34% in 2015. According to Bureau of Labor Statistics Data, the highest recorded monthly unemployment rate (since 1948) was 10.8% in November and December in 1982. With  $H(\delta) = 34\%$  and  $u = 10.8\%$ ,  $\frac{2H(\delta)}{u}(\frac{1}{u} - 1)$  should exceed 52, far above one. In other words, the smallest value of  $\frac{2H(\delta)}{u}(\frac{1}{u} - 1)$  exceeds 52 during 1948-2015. In fact, the inequality still holds even if the share of high-school graduate or below  $H(\delta)$  is as low as 5% and the unemployment rate is as high as 25%. Following the current decreasing trend in the share of high-school graduate or below (minus 1% each year), we expect that the condition holds not only during 1948-2015 but also the coming thirty years or even more. In other words, the fourth property in Theorem 1, together with the other properties, is expected to hold in the United States during 1948-2045.

Up to this point, this paper satisfies its first objective by constructing a model that explains the *three seemingly unrelated* features of unemployment. We should emphasize that our simple modification is indeed widely applicable; however, this paper purposely presents a simple search equilibrium model to shed light on the crucial role of the relativity of search intensity during a job search process. Our theory, though simple, explains the *three seemingly unrelated* features. We proceed to explore the prediction of our theory in its magnitudes in the next section.

## 4 Model Implications

The objective of this section is twofold: first, to evaluate the predictive power of our model, where only the relativity of search intensity matters in determining a job-finding rate; and second, to discuss the implications of this model.<sup>18</sup> In particular, this section derives and evaluates the formulas of the unemployment distribution (10), the OUR-EUR relationship (11), and the fundamental frictional unemployment rate. Throughout this section, the formulas are evaluated using the United States Current Population Survey 1994-2015, and the samples of examination are restricted to the labor force aged 25-60.

In practice, it is difficult to observe the exact human capital value of a worker. We therefore make two assumptions:

**Assumption 1.** *Human capital strictly increases with educational attainment.*

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<sup>18</sup>In the Online Appendix B, we show that the search and matching model, in which the search intensity level matters in determining a job-finding rate, fails in capturing at least two sets of the observations, still less in predicting its magnitudes.

**Assumption 2.** *A human capital distribution is continuous over its support.*

We use  $j$  to denote an educational group: the higher the number of  $j$  the higher is the educational level. Denote  $\bar{\delta}_j$  and  $\underline{\delta}_j$  as the highest and the lowest human capital level in an educational group  $j$ . Denote  $H_j$  as the cumulative distribution function of a worker of type  $\bar{\delta}_j$  in educational group  $j$ . Assumption 1 implies that  $\underline{\delta}_{j+1} > \bar{\delta}_j$ , and Assumption 2 ensures that given any  $j$ ,  $|H(\underline{\delta}_{j+1}) - H(\bar{\delta}_j)| < \varepsilon$  for all  $\varepsilon > 0$ .

## 4.1 The Unemployment Distribution

This subsection discusses the unemployment distribution derived from the model. According to equation (10),  $G(\delta)$  is the cumulative distribution function of human capital in the unemployment, which is indeed the unemployment distribution across educational group. Using equations (10) and (11), simple algebra gives the following theorem.

**Theorem 2. (Unemployment Distribution)** *Suppose the assumption 1 and 2 are satisfied. In a steady state Nash equilibrium, the cumulative distribution function of the educational group  $j$  in the unemployment  $G_j$  is given by*

$$G_j = \frac{1}{2} \frac{\Psi(u)}{\Psi(u_j)}$$

where  $\frac{\Psi(u)}{\Psi(u_j)}$  is the unemployment odds ratio;

$$\Psi(u) \equiv \frac{u}{1-u} \text{ is the overall unemployment odds;}$$

$$\Psi(u_j) \equiv \frac{u_j}{1-u_j} \text{ is the educational unemployment odds } j;$$

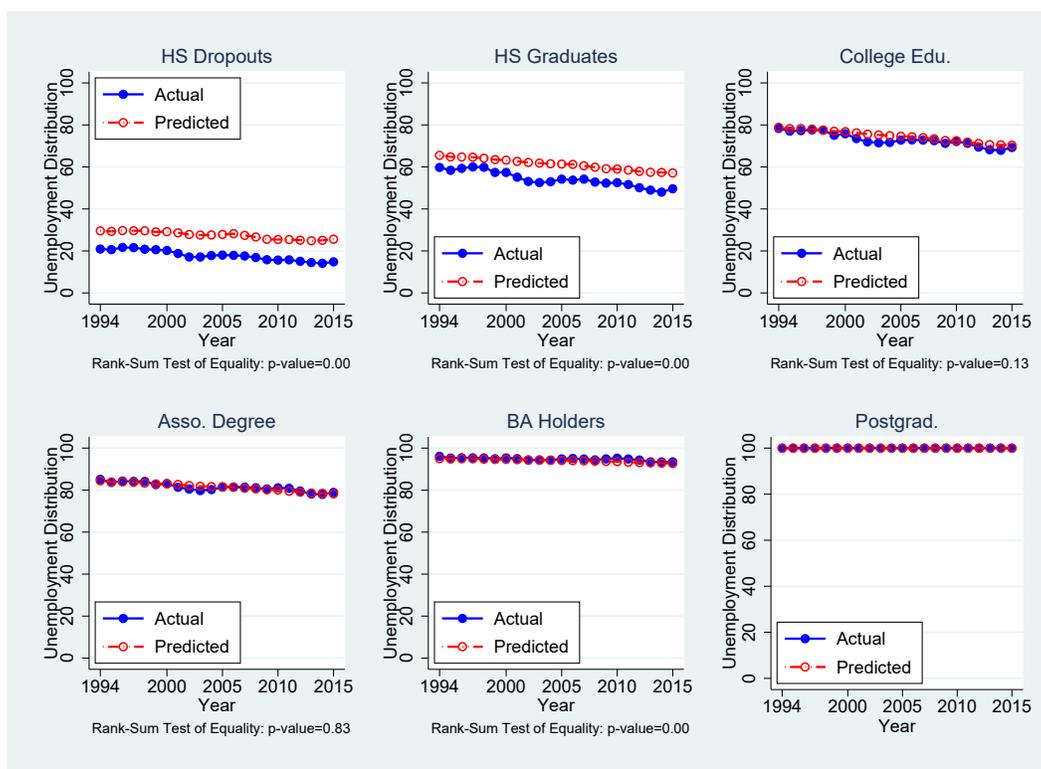
and  $u_j = \left(1 + \frac{4(1-u)H_j}{u^2}\right)^{-\frac{1}{2}}$ .

*Proof.* See the Appendix 6.5. □

Our model derives a novel formula of the unemployment distribution across educational levels, which equals half the unemployment odds ratio. It is noteworthy that  $G(\delta)$  is endogenized in our model; no further restriction is imposed on the function  $G(\delta)$ . To be a valid cumulative distribution function,  $G(\delta)$  has to satisfy three properties: (i) its increases with  $\delta$ , (ii) it equals zero at its lower support, and (iii) it equals one at its upper support. First, differentiating  $G(\delta)$  with respect to  $\delta$ , simple algebra shows that  $G(\delta)$  strictly increases with  $\delta$ . Second, it is straightforward to verify that  $G(\delta)$  approaches zero (one) when  $H(\delta)$  approaches zero (one). We can therefore conclude that  $G(\delta)$  is a valid cumulative distribution function.

Next, we proceed to verify the predictive power of  $G_j$ . According to Theorem 2, the unemployment odds ratio is a function of two variables: the OUR  $u$  and the distribution

Figure 4: Evaluation of the Unemployment Distribution



Notes: Data are from the US CPS. Samples are restricted to the labor force aged 25-60. The predicted data (the dotted line) are generated from Theorem 2.

Table 1: Wilcoxon Rank-Sum Test of Equality between the Actual and the Predicted Unemployment Distribution

State	Unemployment Distribution			
	(1)	(2)	(3)	(4)
	HS Grad.	Some College	AS Degree	BA Holders
AK	0.00**	0.01**	0.00**	0.00**
AL	0.00**	0.17	0.00**	0.00**
AR	0.03**	0.06	0.00**	0.00**
AZ	0.00**	0.01**	0.23	0.81
CA	0.00**	0.00**	0.05**	0.21
CO	0.00**	0.00**	0.00**	0.04**
CT	0.00**	0.00**	0.04**	0.89
DE	0.00**	0.44	0.93	0.04**
FL	0.00**	0.00**	0.03**	0.98
GA	0.00**	0.21	0.42	0.00**
HI	0.00**	0.06	0.72	0.31
IA	0.00**	0.93	0.02**	0.17
ID	0.00**	0.03**	0.44	0.36
IL	0.00**	0.36	0.87	0.09
IN	0.00**	0.42	0.01**	0.00**
KS	0.00**	0.12	0.41	0.96
KY	0.00**	0.66	0.00**	0.00**
LA	0.17	0.01**	0.00**	0.00**
MA	0.00**	0.00**	0.01**	0.96
MD	0.00**	0.37	0.69	0.09
ME	0.00**	0.51	0.72	0.57
MI	0.00**	0.16	0.00**	0.00**
MN	0.00**	0.00**	0.03**	0.64
MO	0.00**	0.45	0.31	0.00**

Notes: p-values are reported. Significance levels: \*\*=5%. HS Grad., Some College, AS Degree, and BA Holders denote high-school graduates, workers with some college education, associate degree holders, and bachelor's degree holders.

Table 3: Wilcoxon Rank-Sum Test of Equality between the Actual and the Predicted Unemployment Distribution (Cont.)

State	Unemployment Distribution			
	(1)	(2)	(3)	(4)
	HS Grad.	Some College	AS Degree	BA Holders
MS	0.00**	0.24	0.00**	0.00**
MT	0.00**	0.00**	0.96	0.96
NC	0.00**	0.83	0.1	0.04**
ND	0.00**	0.61	0.14	0.53
NE	0.00**	0.31	0.71	0.61
NH	0.00**	0.00**	0.00**	0.01**
NJ	0.00**	0.08	0.27	0.09
NM	0.01**	0.57	0.17	0.02**
NV	0.00**	0.01**	0.62	0.67
NY	0.00**	0.01**	0.05**	0.00**
OH	0.00**	1	0.03**	0.00**
OK	0.00**	0.53	0.05**	0.00**
OR	0.00**	0.12	0.57	0.06
PA	0.00**	0.02**	0.72	0.19
RI	0.00**	0.41	0.27	0.03**
SC	0.00**	0.48	0.00**	0.00**
SD	0.00**	0.02**	0.01**	0.78
TN	0.01**	0.5	0.02**	0.00**
TX	0.00**	0.13	0.89	0.02**
UT	0.00**	0.00**	0.17	0.74
VA	0.00**	0.01**	0.2	0.59
VT	0.00**	0.00**	0.04**	0.37
WA	0.00**	0.00**	0.12	0.2
WI	0.00**	0.67	0.19	0.00**
WV	0.07	0.15	0.00**	0.00**
WY	0.00**	0.03**	0.76	0.15

Notes: p-values are reported. Significance levels: \*\*=5%. HS Grad., Some College, AS Degree, and BA Holders denote high-school graduates, workers with some college education, associate degree holders, and bachelor's degree holders.

function of educational level  $H_j$ , which are easily accessible. Using the US CPS, we compute the yearly  $u$  and  $H_j$  for each educational group. Both the actual (the solid lines) and the predicted (the dash lines) annual  $G_j$  are plotted in Figure 4. Indeed, the predicted  $G_t$  are about 10 percent and five percent more than the actual ones for high-school dropouts and high-school graduates. One of the possibilities, accounting for the difference, is that low-educated unemployed workers might decide not only on their search intensity, but also on whether to participate to the labor force. Nevertheless, our model did not explicitly endogenize the labor force participation decision and we leave it for future research avenue.<sup>19</sup> We perform the Wilcoxon rank-sum test of equality in each educational group. Surprisingly, the null hypothesis that the actual and the predicted  $G_j$  are from an identical distribution cannot be rejected at any conventional significance level for workers with some college education and associate degree holders. Although the rank-sum test rejects the equality between the actual and the predicted  $G_t$  for bachelor's degree holders, they are so close that the two lines overlap with each other during the entire period of examination. Regarding postgraduates, the predicted  $G_t$  is always equal to one because their human capital level is assumed to be the highest in the economy due to Assumption 1. With certainty, the two lines are identical in this educational category.

We conduct the same exercise in each of the 50 states in the United States, and plot the actual and the predicted values in Online Appendix D. The corresponding p-values are reported in Table 1. Surprisingly, the predictability of Theorem 2 in each state is as good as its performance in an economy as a whole. Over 50 percent of the states, the null hypothesis on the equality between the two sets of values cannot be rejected for some college education, associate degree holders, and bachelor's degree holders. We conclude that Theorem 2 succeeds in predicting the unemployment distribution in magnitude for workers with some college education, associate degree holders, and bachelor's degree holders.

## 4.2 The Disaggregation Theory

This subsection purposes to derive a formula that allows economists, policymakers, and the public to disaggregate the OUR into various EURs. Indeed, equation (11) is the disaggregation formula that maps an OUR to the unemployment rate of the  $x$ th percentile of a human capital distribution. Although  $x$  is unobservable, equation (11) can be computed under Assumption 1 and 2 as follows.

**Theorem 3. (*Disaggregation Theory*)** *Suppose the assumption 1 and 2 are satisfied. In a*

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<sup>19</sup>For example, incorporating the search relativity in the framework of Alvarez and Shimer (2011) could be a fruitful and interesting research area.

steady state Nash equilibrium, the EUR of the educational group  $j$  is given by

$$u_j = \frac{2u}{(B_j + B_{j-1})} \quad (17)$$

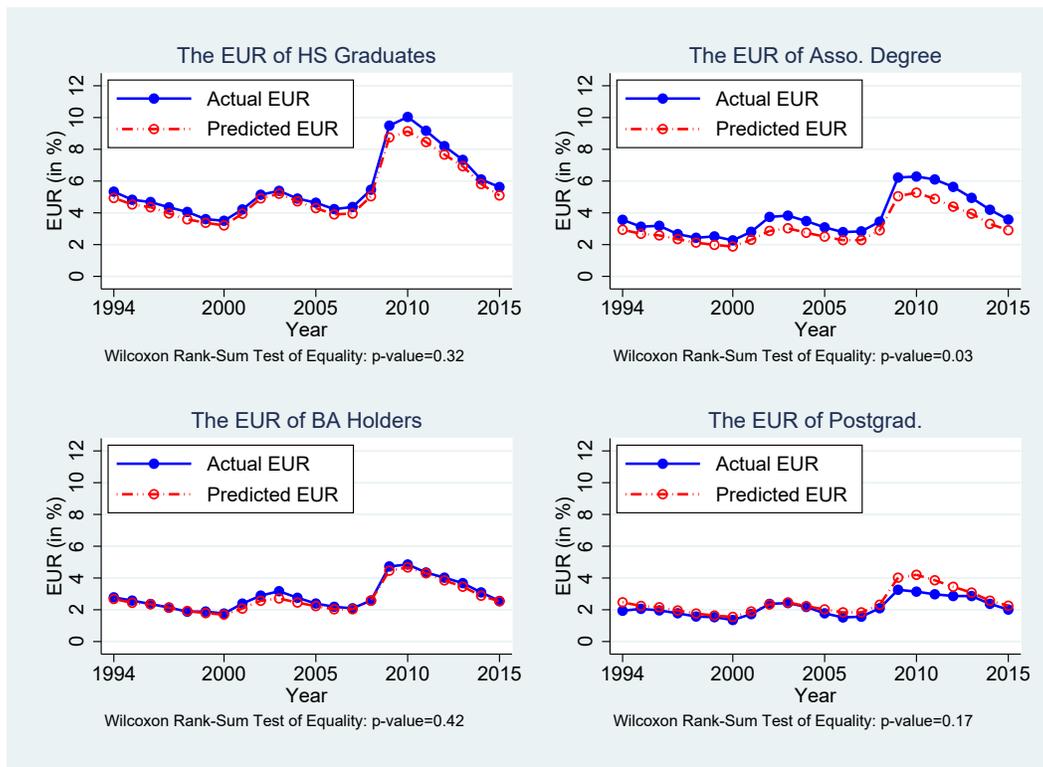
where  $B_j \equiv (u^2 + 4(1 - u)H_j)^{\frac{1}{2}}$

*Proof.* See the Appendix 6.6. □

Using the US CPS, the actual and the predicted EURs of high-school graduates, associated degree holders, bachelor’s degree holders, and postgraduates are computed. Both the actual (the solid lines) and the predicted (the dash lines) annual unemployment rates of 1994-2015 are plotted in Figure 5. As expected due to Theorem 1, the two sets of values display a similar trend regardless educational level. More strikingly, the magnitude of the actual and the predicted EURs are so close that the null hypothesis that they are drawn from an identical distribution cannot be rejected by the Wilcoxon rank-sum test for the high-school graduates, the bachelor’s degree holders, and the postgraduates. Certainly, the disaggregation theory does a superb performance in mapping the OUR into the EURs in the United States.

To further evaluate the predictability of Theorem 3, we apply the disaggregation theory to predict the yearly EURs of high-school graduates, associated degree holders, bachelor’s degree holders, and postgraduates by state during 1994-2015. Both the actual (the solid lines) and the predicted (the dash lines) annual unemployment rates of 1994-2015 are plotted in Online Appendix E. According to the figure, the observed and the predicted EURs are close in its trend and magnitude for most the educational groups and states. We perform the Wilcoxon rank-sum test for each of the educational groups in each state and report the corresponding p-values in Table 2. The table suggests that most the tests cannot reject the null hypothesis that the actual and the predicted EURs are from an identical distribution. Table 3 summarizes the results of the Wilcoxon rank-sum test. According to Table 3, the null hypothesis cannot be rejected at one percent level in all of the 50 states for the high-school graduates. Meanwhile, we cannot reject the null hypothesis in 40, 48, and 39 out of 50 states for associate degree holders, bachelor’s degree holders, and postgraduates at one percent significance level. At the conventional significance level (five percent), we cannot reject the null hypothesis in over half the United States regardless of educational level. Surprisingly, the null hypothesis cannot be rejected over 50 percent of the states for high-school graduates (90 percent), bachelor’s degree holders (84 percent), and postgraduates (66 percent) at 10 percent significance level. More strikingly, in the group of high-school graduates and bachelor’s degree holders, the null hypothesis cannot be rejected in about half the United States at 50 percent significance level. These results suggest that the derived formula (3) performs so well (if not considered as nearly perfectly) in predicting both

Figure 5: Evaluation of the Disaggregation Theory



Notes: Data are from the US CPS. Samples are restricted to the labor force aged 25-60. The predicted data (the dotted line) are generated from Theorem 3.

Table 2: Wilcoxon Rank-Sum Test of Equality between the Actual and the Predicted EURs

State	Disaggregation			Approximation	
	(1)	(2)	(3)	(4)	(5)
	HS Grad.	AS Degree	BA Holders	Postgrad.	Postgrad.
AK	0.02**	0.00**	0.01**	0.00**	0.00**
AL	0.29	0.03**	0.23	0.00**	0.00**
AR	0.04**	0.71	0.13	0.00**	0.00**
AZ	0.47	0.05	0.32	0.72	0.85
CA	0.16	0.04**	0.21	0.53	0.69
CO	0.76	0.15	0.09	0.21	0.15
CT	0.94	0.28	0.26	1.00	0.81
DE	0.34	0.32	0.85	0.27	0.37
FL	0.66	0.05**	0.26	0.94	0.81
GA	0.30	0.02**	0.66	0.05**	0.10
HI	0.27	0.20	0.98	0.45	0.51
IA	0.74	0.32	0.41	0.24	0.28
ID	0.85	0.03**	0.89	0.25	0.29
IL	0.32	0.06	0.83	0.21	0.37
IN	0.69	0.24	0.74	0.03**	0.03**
KS	0.57	0.08	0.34	0.56	0.89
KY	0.22	0.01**	0.27	0.00**	0.00**
LA	0.08	0.00**	0.00**	0.00**	0.00**
MA	0.89	0.16	0.13	0.93	0.64
MD	0.36	0.32	0.67	0.32	0.53
ME	0.54	0.28	0.89	0.22	0.34
MI	0.67	0.29	0.64	0.01**	0.01**
MN	0.71	0.04**	0.25	0.61	0.40
MO	0.89	0.08	0.93	0.00**	0.01**

Notes: p-values are reported. Significance levels: \*\*=5%. HS Grad., AS Degree, BA Holders, Postgrad. denote high-school graduates, associate degree holders, bachelor's degree holders, and postgraduates. Column (1)-(4) uses Theorem 3, and column (5) uses Theorem 4.

Table 4: Wilcoxon Rank-Sum Test of Equality between the Actual and the Predicted EURs (cont.)

State	Disaggregation			Approximation	
	(1)	(2)	(3)	(4)	(5)
	HS Grad.	AS Degree	BA Holders	Postgrad.	Postgrad.
MS	0.04**	0.10	0.02**	0.00**	0.00**
MT	0.94	0.00**	0.81	0.76	0.93
NC	0.37	0.57	0.64	0.27	0.35
ND	0.93	0.02**	0.26	0.94	0.93
NE	0.98	0.11	0.94	0.72	1.00
NH	0.51	0.01**	0.02**	0.20	0.10
NJ	0.62	0.08	0.26	0.32	0.43
NM	0.18	0.02**	0.78	0.02**	0.04**
NV	0.32	0.05**	0.53	0.80	0.89
NY	0.59	0.04**	0.03**	0.19	0.39
OH	0.56	0.21	0.98	0.03**	0.05**
OK	0.34	0.01**	0.91	0.01**	0.01**
OR	0.64	0.04**	0.61	0.14	0.56
PA	0.32	0.00**	0.72	0.53	0.64
RI	0.42	0.08	0.71	0.15	0.21
SC	0.37	0.49	0.22	0.00**	0.00**
SD	0.51	0.64	0.04**	0.51	0.62
TN	0.15	0.17	0.50	0.03**	0.05**
TX	0.02	0.01**	0.81	0.20	0.29
UT	0.96	0.29	0.27	0.79	0.67
VA	0.85	0.04**	0.34	0.83	0.57
VT	0.98	0.32	0.11	0.80	0.89
WA	0.94	0.01**	0.23	0.20	0.36
WI	0.91	0.21	0.78	0.09	0.11
WV	0.13	0.01**	0.04**	0.00**	0.00**
WY	0.76	0.02**	0.87	0.21	0.24

Notes: p-values are reported. Significance levels: \*\*=5%. HS Grad., AS Degree, BA Holders, Postgrad. denote high-school graduates, associate degree holders, bachelor's degree holders, and postgraduates. Column (1)-(4) uses Theorem 3, and column (5) uses Theorem 4.

Table 3: Summary of the Wilcoxon Rank-Sum Test of Equality

Significance	Disaggregation			Approximation	
Level	(1)	(2)	(3)	(4)	(5)
	HS Grad.	AS Degree	BA Holders	Postgrad.	Postgrad.
1%	50 (100)	40 (80)	48 (96)	39 (78)	39 (78)
5%	46 (92)	26 (52)	43 (86)	34 (68)	35 (70)
10%	45 (90)	20 (40)	42 (84)	33 (66)	33 (66)
25%	40 (80)	12 (24)	34 (68)	21 (42)	29 (58)
50%	27 (54)	3 (6)	23 (46)	16 (32)	18 (36)

Notes: There are 50 states in the United States. DC is excluded. The number reported in this table is the number of states that fails to reject the null hypothesis that the observed and the predicted EURs are from an identical distribution at the corresponding significance level. The number in parentheses is the percent of the total number of the states, in which the null hypothesis that the observed and the predicted EURs are from an identical distribution cannot be rejected at the corresponding significance level. HS Grad., AS Degree, BA Holders, Postgrad. are high-school graduates, associate degree holders, bachelor’s degree holders, and postgraduates. Column (1)-(4) uses the disaggregation theory 3, and column (5) uses the disaggregation approximation theory 4.

the trends and the magnitudes of the EURs that most the statistical tests could not reject the null hypothesis that the derived formula (3) is identical to the underlying data generating process of the EURs.

**Remarks.** Thus far, we have demonstrated the superb performance of Theorem 2 and 3. Before closing this subsection, several points deserve attention. First, our theories utilize the least number of input variables. The unemployment distribution and the EUR are functions of only two variables: an OUR  $u$  and a cumulative distribution function  $H_j$ , free from other parameters. The degree of freedom is indeed zero. Consider two formulas with different numbers of input variables. If both the formulas could predict any one of the unemployment distribution and the EUR at the same accuracy level, it is likely that the formula with less number of input variables is preferred.

Of course, the accessibility of input variables matters in the selection of any two formulas. Our theory requires the overall unemployment rate and the fraction of each educational group. Needless to mention, it is straightforward to obtain the OUR and the distribution of the educational attainment in most (if not all) countries. On the contrary, it is rather challenging to obtain the information about individual search intensity level, the functional form of a matching function, and a separation rate  $\lambda$ . Therefore, given the same accuracy, our formulas have its advantage over others (if others exist) in the accessibility of its inputs.

Third, the two formulas are not computer-intensive. Unlike nowadays fancy estimation methods, our formulas involve no recursive structure, systems of equations, or samples of an enormous number of observations. Indeed, the computation takes almost no time and is easy to implement for economists, policymakers, and the public.

Of course, with these three advantages, the predictability of the formulas is our concern. As demonstrated, the Wilcoxon rank-sum test of equality cannot reject most the null hypotheses that the actual and the observed values are drawn from the same distribution at any conventional significance level in almost all the states. With no doubt, we can conclude that our model describes well the macroeconomic relationship between the OUR and the unemployment distribution and between the OUR and each EUR and the underlying theory succeeds in mapping the OUR into the unemployment distribution and each EUR in at least four aspects: its predictability, its least number of input variables, its accessibility of the required inputs, and its easiness of implementation.<sup>20</sup>

### 4.3 Fundamental Frictional Unemployment Rate

In this section, we discuss a measure of the fundamental frictional unemployment rate (FFUR). To begin with, recall that the unemployment rate of workers falls with their average search intensity. This subsection asks if workers keep increasing their search intensity and/or human capital, does their unemployment rate approach zero? In other words, if search cost is sufficiently low and human capital is sufficiently high, does search frictional unemployment vanish? If not, what is the frictional unemployment rate, abstracting search effort and human capital?

Here, we define the FFUR as the unemployment rate that associates with the unemployment spell that cannot be shortened by further increasing one's search effort or human capital. Therefore, there exists no EUR that can be lower than the FFUR of an economy. Lemma 1 implies that those with the highest human capital level should search the most intensively. It is therefore expected that workers with the highest human capital level ranks top in the search ladder. That is,  $F(s) = 1$ , leaving their transition rate equal to  $p$ . With the highest human capital level in the economy, their unemployment rate in principle sheerly arises from search friction, net of search effort and human capital. Workers with more human capital experience a shorter unemployment spell because they choose to search more intensively. Those, who possess the highest human capital level, place themselves on the top of the search ladder. An extra search intensity does not allow the workers to surpass others in the search game. Their unemployment rate remains at the FFUR even though their human capital and search intensity further increase.

Using equation (11), when  $H(\delta)$  approaches one, the FFUR is equivalent to  $u/(2-u)$ . The FFUR can be interpreted as the efficiency of the matching technology in one country's labor market. The lower is the rate, the more efficient is the matching technology. Truly, to conclude whether a higher natural rate of unemployment is attributable to the inefficient

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<sup>20</sup>The authors also verified that the formula works well using Canadian Labour Force Survey 1994-2015 and the United Kingdom Labour Force Survey 1994-2015. Testing the implications of the search relativity model in other OECD countries will definitely be a fruitful research avenue.

matching technology or the workers' lower search effect is rather hasty. The derived FFUR plays its role in establishing a positive relation between the OUR and the efficiency of a matching technology. As the fundamental frictional unemployment rate strictly increases with  $u$ , the derived FFUR confirms that a lower natural rate of unemployment does reflect a more efficient matching technology in a labor market. More importantly, using  $u/(2-u)$ , the magnitude of FFUR, though unobservable, can be quantified.

In fact, if the share of the educational group  $j$  with the highest human capital level is small enough,  $H_j$  approaches one. If Assumption 1 is satisfied, the postgraduate workers are the ones with the highest human capital level. With the following assumption, the EUR of postgraduates is approximately equal to the FFUR.

**Assumption 3.** *The share of the postgraduate workers is sufficiently small.*

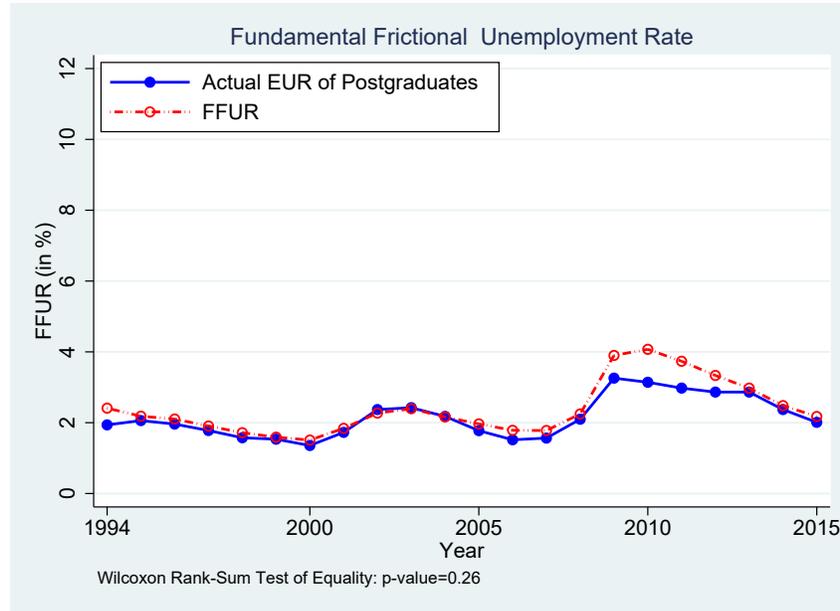
**Theorem 4. (Disaggregation Approximation Theory)** *Suppose the assumption 1 and 3 are satisfied. In a steady state Nash equilibrium, the unemployment rate of the postgraduates is given by*

$$u_{Postgraduates} \approx \lim_{H(\delta) \rightarrow 1} \left( 1 + \frac{4(1-u)H(\delta)}{u^2} \right)^{-\frac{1}{2}} = \frac{u}{2-u} \quad (18)$$

The disaggregation approximation theory shows that the unemployment rate of the postgraduates is a function of an OUR only. We evaluate this disaggregation approximation theory by comparing the FFUR with the EUR of postgraduates using the US CPS. If the two set of values are different, we should cast doubt on our argument in this subsection. Figure 6 shows that the actual EUR of postgraduates and the FFUR exhibit a similar trend both in recession and expansion. The two sets of values are so close that they overlap for ten consecutive years during 1994-2004. However, the FFUR are slightly higher than the actual EUR in slumps during 2008-2010. With the exception of the fair performance in slumps, the approximation method basically performs well in capturing the EUR of the postgraduates. We again simulate the FFUR and the EUR of postgraduates by state and the figure is presented in Online Appendix F. Also, we perform the Wilcoxon rank-sum test of equality and the corresponding p-values are reported in Table 2. 3 summarizes the results of the Wilcoxon rank-sum test. This exercise is astonishing because the null hypothesis that the two sets of values are from an identical distribution cannot be rejected in 70 percent of the States at five percent significance level. We cannot reject the hypothesis in over half the United States at 25 significance level. With only one input variable, the predictive power of the disaggregation approximation theory is exceptionally high. This approximation theory preserves all the advantages of the disaggregation theory: its predictability, its least number of input variables, its accessibility of the required input, and its easiness of implementation. Nevertheless, the drawback of this theory is that it can only be applied to the postgraduates,

not the other educational categories.

Figure 6: Evaluation of Fundamental Frictional Unemployment Rate



Notes: Data are from the US CPS. Samples are restricted to the labor force aged 25-60. The predicted data (the dotted line) are generated from Theorem 4.

This FFUR is derived by assuming that only the relativity of search intensity matters in determining a transition rate. Suppose both the level and the relativity of search intensity are also the determinants of the transition rate. The unemployment spell for the postgraduates is expected to be shorter than the one associated with FFUR; the EUR of the postgraduates in principle should be far below the FFUR. If an unemployment rate is solely driven by search friction, the distance between the FFUR and the EUR of postgraduates should be attributable to the level of search intensity. According to Figure 6, the two sets of values are so close that the statistical test could not reject the null hypothesis that the derived formula is identical to the underlying data generating process of the EURs at any conventional significance level. It basically leaves almost no room that can be explained by the level of search intensity. But the actual EURs are indeed lower than the FFUR in slumps. Of course, one could argue that parts of the gap is attributed to the level of search intensity. Consider that the existing search equilibrium model, where the level of search intensity is one of the determinants of the job-finding rate, could not capture more than one documented unemployment trend, as shown in the Online Appendix B. We cast doubt on the possibility that the gap between the FFUR and the EUR is explainable by the level of search intensity. In fact, one of the possibilities is that the unemployment rate of this period is not only

generated by search friction, but also by rationing (Michaillat, 2012) and ambiguity (Chan and Yip, 2017), which are shown to play a crucial role in unemployment in economic downturns. We therefore conclude that while the relativity of search intensity is the key determinants of the transition rate, the possibility that the level of search intensity matters in determining the transition rate is at best low.

## 5 Conclusion

This article documents the *three seemingly unrelated* features of unemployment, in which the existing search and matching equilibrium model cannot capture if the search intensity level determines a job-finding rate. We construct a model that only the relative position of search intensity matters in determining the transition rate. We show that our model could explain all the documented features of unemployment.

Our model derives three novel formulas for the unemployment distribution, the EURs, and the fundamental frictional unemployment rate. Our numerical exercises suggest that the formulas predict the magnitude of the corresponding variables so well that the null hypothesis in which the actual and the predicted values of the variables are identical cannot be rejected at any conventional significance level in most cases.

This paper brings a new research direction in several aspects. First, this paper succeeds in disaggregating the OUR into the EURs of various education groups. It will be fruitful and interesting to disaggregate the overall unemployment duration into the unemployment duration by educational attainment. Second, with the disaggregation theory, it is interesting to also compute and explore the properties of the elasticity of EURs with respect to OUR. With no doubt, estimating such elasticity would be a new and interesting research area. Third, the proposed theory embeds the search relativity in the undirected search model. Incorporating the relativity in a directed search model with on-the-job search allows the model to also predict the on-the-job transition rate. Such generalization completes our understanding in the job-seeking behaviours of not only the unemployed but also the employed.

## 6 Appendix: Proof

### 6.1 The Derivation of Equation (10)

Using equation (6),  $G(\delta)$  is the solution of the following differential equation.

$$G'(\delta)u = \frac{\lambda h(\delta)}{\lambda + G(\delta)p} \quad (19)$$

Notice that  $u$  is shown to be independent of  $\delta$ . Rearranging terms, we have

$$(\lambda + G(\delta)p)G'(\delta)u = \lambda h(\delta)$$

Integrating both side from  $z$  to  $x$ , we have

$$\begin{aligned} u \int_z^x \lambda + G(\delta)p dG(\delta) &= \int_z^x \lambda h(\delta) d\delta \\ u\lambda G(\delta) + up \int_z^x G(\delta) dG(\delta) &= \lambda H(x) \\ up \frac{G^2(\delta)}{2} &= \lambda(H(\delta) - G(\delta)u) \end{aligned}$$

Solving the quadratic equation gives the solution of  $G(\delta)$  as in equation (10).

## 6.2 The Derivation of Equation (14)

Using  $F(s^*(\delta)) = G(\delta)$  and equation (10),

$$F(s^*(\delta)) \frac{\beta p}{r + \lambda} = \frac{\beta \lambda}{r + \lambda} (\Phi_1(\delta) - 1) \quad (20)$$

where  $\Phi_1(\delta) = \sqrt{1 + \frac{2pH(\delta)}{\lambda u}}$ .

Using equation (8) and (20), we have

$$\delta - rJ^U(\delta) = \frac{\delta + C(s^*(\delta)) - z}{1 + \Phi_2(\delta)}$$

where  $\Phi_2(\delta) = \frac{\beta \lambda (\Phi_1(\delta) - 1)}{r + \lambda}$ . Substituting the above equation,  $f(s^*(\delta)) ds^*(\delta)/d\delta = g(\delta)$  and equation (11) into equation (12), we have

$$C'(s^*(\delta)) \frac{ds^*(\delta)}{d\delta} = \frac{h(\delta)}{u\Phi_1(\delta)} \frac{\beta p}{r + \lambda} \frac{\delta + C(s^*(\delta)) - z}{1 + \Phi_2(\delta)}$$

which can be written as equation (14).

## 6.3 Proofs of the Properties of $\Omega(\delta, p)$

$$T(\delta) = \frac{\lambda \beta h(\delta) A}{\beta \lambda (1 + 2AH(\delta)) + (r + \lambda - \beta \lambda) \sqrt{1 + 2AH(\delta)}}$$

where  $A = 2(1-u)/u^2$ . It is straightforward to show that  $\lim_{p \rightarrow 0} T(\delta) = 0$  and  $\lim_{p \rightarrow \infty} T(\delta)$  is finite. Hence,  $\lim_{p \rightarrow 0} \Omega(\delta, p) = 0$  and  $\lim_{p \rightarrow \infty} \Omega(\delta, p)$  is positive finite. Differentiating

$T(\delta)$  with respect to  $p$ , we have

$$\frac{\partial T(\delta)}{\partial p} = \frac{2-u}{u^2} \frac{\partial T(\delta)}{\partial A}$$

One could easily verify that  $\partial T(\delta)/\partial p > 0$ ,  $\lim_{p \rightarrow \infty} \frac{\partial T(\delta)}{\partial p} = 0$ . Differentiating  $\Omega(\delta, p)$  with respect to  $p$ , we have

$$\frac{\partial \Omega(\delta, p)}{\partial p} = \int_z^\delta \frac{\partial T(x')}{\partial p} (x' - z) e^{\int_{x'}^\infty T(x) dx} dx' + \int_z^\infty \left( \int_{x'}^\infty \frac{\partial T(x)}{\partial p} dx \right) T(x') (x' - z) e^{\int_{x'}^\infty T(x) dx} dx'$$

Hence, we have  $\partial \Omega(\delta, p)/\partial p \geq 0$  and  $\lim_{p \rightarrow \infty} \partial \Omega(\delta, p)/\partial p = 0$ .

## 6.4 Proof of Theorem 1

Differentiating  $u_\delta$  in equation (11) with respect to  $\delta$ , we have

$$\frac{du_\delta}{d\delta} = -\frac{2(1-u)h(\delta)}{u^2} \left( 1 + \frac{4(1-u)H(\delta)}{u^2} \right)^{-\frac{3}{2}} < 0$$

Differentiating the above derivative with respect to  $p$ , we have

$$\frac{d^2 u_\delta}{dud\delta} = -\frac{u + 2(1-u)}{u(1-u)} \frac{2H(\delta)(1-u) - u^2}{u^2 + 4(1-u)H(u)}$$

$d^2 u_\delta / dud\delta > 0$  iff  $2H(\delta)(1-u) < u^2$ .

## 6.5 Proof of Theorem 2

Using equations (10) and (11), we have

$$\begin{aligned} G(\delta) &= \frac{u}{2(1-u)} \left( \sqrt{1 + \frac{4(1-u)H(\delta)}{u^2}} - 1 \right) \\ &= \frac{1}{2} \frac{u}{1-u} \left( \frac{1}{u_\delta} - 1 \right) \\ &= \frac{1}{2} \frac{\Psi}{\Psi} \frac{1-u_\delta}{u_\delta} \\ &= \frac{1}{2} \frac{\Psi}{\Psi_\delta} \\ &= \frac{1}{2} \Theta_\delta \end{aligned}$$

Assumption 1 and 2 imply that  $\underline{\delta}_j > \bar{\delta}_{j-1}$ . Hence, we have

$$G_j = \frac{1}{2}\Theta_j$$

## 6.6 Proof of Theorem 3

$$\begin{aligned}
u_j &= \frac{\int_{\underline{\delta}_j}^{\bar{\delta}_j} \left(1 + \frac{4(1-u)H(\delta)}{u^2}\right)^{\frac{-1}{2}} h(\delta) d\delta}{H_j - H_{j-1}} \\
&= \frac{\int_{\underline{\delta}_j}^{\bar{\delta}_j} \left(1 + \frac{4(1-u)H(\delta)}{u^2}\right)^{\frac{-1}{2}} dH(\delta)}{H_j - H_{j-1}} \\
&= \frac{\int_{H_{j-1}}^{H_j} \left(1 + \frac{4(1-u)x}{u^2}\right)^{\frac{-1}{2}} dx}{H_j - H_{j-1}} \\
&= \frac{u}{H_j - H_{j-1}} \int_{H_{j-1}}^{H_j} (u^2 + 4(1-u)x)^{\frac{-1}{2}} dx \\
&= \frac{u}{H_j - H_{j-1}} \frac{1}{4(1-u)} \int_{B_{j-1}}^{B_j} y^{\frac{-1}{2}} dy \\
&= \frac{u}{H_j - H_{j-1}} \frac{1}{2(1-u)} y^{\frac{1}{2}} \Big|_{B_{j-1}}^{B_j} \\
&= \frac{2u}{(B_j^{\frac{1}{2}} + B_{j-1}^{\frac{1}{2}})}
\end{aligned}$$

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