THE TRADEOFF BETWEEN EMPLOYMENT AND ENVIRONMENT

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Abstract

How do environmental policies create job losses? We construct a two-sector search equilibrium model to answer the question. We establish conditions under which unemployment is inevitable. We analytically show that if emission tax revenues are not rebated to agents in the labor market, intensity standards dominate simple emission taxes: for any emission tax policy, there always exist intensity standards that lead to higher environmental quality and lower unemployment. Conversely, no emission tax rate dominates any intensity standard. A Pigouvian tax corrects production externality along its intensive margin, not its extensive margin. The distribution of vacancies between polluting and non-polluting sectors can be suboptimal: the creation of vacancies in one sector makes their goods relatively more abundant (less valuable) and the other type of goods more valuable, creating a job allocation externality. Under certain conditions, if emission tax revenues are rebated to firms, revenue-neutral emission taxes internalize these externalities.

Keywords: Unemployment; Intensity Standards; Emission Taxes.

JEL Classification Numbers: D62, H23, J64, Q52, Q58.

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1 Introduction

Environmental policies could improve environmental quality and efficiency by correcting production externality. However, they may create job losses (Greenstone, 2002; Walker, 2013; Yip, 2018, 2019). The tradeoff between environmental quality and labor market outcomes often plays an important role in the political debate over environmental lawmaking.\footnote{For example, the proposal of fuel tax increases led to the “Yellow Vest” in France. In the end, the President of France suspended the increases and pledged €100 per month increase in the minimum wage.} This paper answers why environmental policies often come with unemployment, uncovers major factors that lead to the inevitability of the tradeoff between environmental quality and employment opportunity, and reveals major mechanisms that lead to the welfare tradeoffs of environmental policies in imperfect labor markets.

Model assumptions in the literature often hinder our understanding of how environmental policies trade environmental quality off against employment opportunity. First, the intensive margin of emission is assumed to be fixed in the early literature (Carraro et al., 1996; Bye, 2002), which can be interpreted as a short-term technological constraint. However, this margin is found to be empirically important for controlling emission (Shapiro and Walker, 2018). Meanwhile, the extensive margin is the driving force for job losses upon the introduction of environmental policies (Walker, 2011). Second, real labor markets are imperfect, with frictions, externalities, and rigidities which prevent instant market clearing and efficient labor allocation, consequently creating involuntary unemployment and welfare losses. These features are often omitted in previous studies (e.g. Bovenberg and van der Ploeg (1996) and Bovenberg and van der Ploeg (1998a)). Finally, some previous studies only model one sector and abstract sectoral reallocation of labor (Bovenberg and van der Ploeg, 1998b). However, sectoral reallocation of labor can be an important factor of unemployment and welfare outcomes (Wagner, 2005; Walker, 2013; Hafstead and Williams, 2018).

This paper contributes to the literature by improving on the shortcomings of previous studies. It incorporates endogenous emission decisions along extensive and intensive margins into a two-sector search equilibrium model (Acemoglu, 2001; Chassamboulli and Palivos, 2014). We therefore allow adjustment on emission decisions along both margins. In addition to the emission decisions, our analytically tractable model captures frictional unemployment, sectoral reallocation, and price effects, all of which are important to the problem. Given its stylized nature and analytical tractability, our model helps economists and policymakers understand the major mechanisms through which environmental policies reduce emissions and create job losses.

We utilize this simple and intuitive model to analyze a broad class of policies (Class I). These policies include a lump-sum tax on polluting firms, a simple per-unit emission tax where the tax revenue is not rebated to agents in the labor market, and an intensity
standard. The common feature of this class of policies is (1) it incurs a higher flow cost in the polluting sector, (2) it does not increase emission along the intensive margin, and (3) it does not affect the flow production cost of the nonpolluting sector. In addition, we study the unemployment effect and the efficiency consequence of a revenue-neutral emission tax policy. Our analysis yields four remarkable results.

First, no Class I policy can yield a better environmental quality without creating more involuntary unemployment. A higher flow cost of production in the polluting sector reduces the value and thus the supply of their vacancies. This direct effect raises unemployment and reduces the market share of the polluting sector, leading to the tradeoff. The expansion of the market share of the nonpolluting sector lowers the relative price of their products, depressing the supply of their vacancies. This general equilibrium effect further increases unemployment.

More impressively, our framework allows us to derive two sufficient statistics—the optimal intensive margin of emission and the total flow cost at a firm level—from the partial equilibrium of a firm problem to show the inevitable tradeoff between employment and environmental quality among different types of policies in the search equilibrium framework. In other words, we can establish the inevitability of the tradeoff between environmental quality and employment opportunity between any two policies within Class I without solving the entire structural model. This feature greatly simplifies the analysis and is novel in the literature.

Second, intensity standards always dominate simple emission taxes in the labor market. We show that given any emission tax rate, there always exist intensity standards that lead to a lower unemployment rate and a better environmental quality than the simple emission tax. However, given any intensity standard, there does not exist such an emission tax rate. The reason is that without paying an emission tax, intensity standards incur a lower flow cost than simple emission taxes to achieve the same amount of emission abatement on the firm level. This result applies to every positive intensity standard and every positive emission tax rate. In contrast, while the literature provides various rationales to prefer intensity standards to simple emission taxes in different contexts (Gerlagh and Van der Zwaan, 2006; Fischer and Springborn, 2011; Li and Shi, 2011; Holland, 2012; Parry and Williams, 2012; Tombe and Winter, 2015), only Hafstead and Williams (2018) consider the advantage of intensity standards over emission taxes in the labor market, focusing exclusively on a specific intensity standard and an equivalent revenue-neutral emission tax. Our analytical findings complete the picture: it generalizes the policy comparison in terms of environmental quality and employment opportunity between all possible intensity standards and emission tax rates.

Third, a revenue-neutral emission tax, under certain conditions, reduces emission and depresses unemployment. An emission tax reduces emission along the intensive margin. An increased flow cost on polluting firms raises unemployment and reduces the market
share of the polluting sector (emission along the extensive margin). Hence, the environmental quality is improved along both margins. While the market share of nonpolluting sector increases, the relative price and thus the revenue decrease in the nonpolluting sector. This general equilibrium effect shrinks employment in the nonpolluting sector. Meanwhile, the rebate of emission tax revenue increases the revenue in both sectors, expanding employment. If the elasticity of revenue with respect to its market share is small enough in the nonpolluting sector, the negative general equilibrium effect on their revenue can be outweighed by the direct impact of the rebate. Consequently, the revenue-neutral emission tax scheme reduces both unemployment and emission. We are the first to establish the conditions under which environmental policies could avoid the tradeoff between environmental quality and employment opportunity, providing economists and policymakers with new factors and mechanisms to consider in designing environmental policies.

Fourth, a revenue-neutral emission tax, under certain conditions, is efficient in the labor market. While the production in the polluting sector creates pollution that causes welfare loss (i.e., production externality), the creation of vacancies also creates another externality, job allocation externality, in the labor market. In our search equilibrium framework, the marginal cost of creating a vacancy in either sector is identical. A free-entry condition ensures that the expected profit in creating a vacancy in either sector is identical in a decentralized equilibrium. Yet, firms do not take into account that the creation of a vacancy in one sector expands its market share and reduces the relative price and thus the revenue of each existing position in this sector.

Meanwhile, its creation shrinks the other sector and increases the revenue of all existing positions in that sector. Hence, this job allocation externality leads to an excessively high fraction of either the polluting or the nonpolluting sector in equilibrium. We show that a revenue-neutral emission tax could reap a triple dividend: while a Pigouvian tax on emission corrects the production externality along its intensive margin, the rebate of the tax revenues corrects the production externality along its extensive margin and internalizes the job allocation externality. The second and the third dividend differ from the reduction in the distortion cost of the preexisting tax, suggested in the literature on the double-dividend of a revenue-neutral emission tax scheme (Goulder, 1995; Parry and Bento, 2000; Chiroleu-Assouline and Fodha, 2006; Bento and Jacobsen, 2007; Chiroleu-Assouline and Fodha, 2014; Gahvari, 2014). To the best of our knowledge, it is the first paper to consider recycling emission tax revenues to internalize the job allocation externality to improve social welfare.

This paper complements a series of influential papers by Bovenberg and van der Ploeg that study the unemployment effects of environmental policies. This paper differs from Bovenberg and van der Ploeg (1996) and Bovenberg and van der Ploeg (1998a) by mod-

\[^{2}\]In a perfectly competitive model, firms equate marginal revenue to marginal cost, and the free-entry condition equalizes profits in all sectors in a decentralized equilibrium.
eling the underlying labor market imperfection that leads to involuntary unemployment. Bovenberg and van der Ploeg (1998b), integrating frictional unemployment, study the unemployment effects of environmental taxes. While these models, with only one formal sector, abstract sectoral reallocation, our model features multiple sectors, allowing nonpolluting sectors to absorb unemployed workers from polluting sectors. This mechanism is shown to be important in recent works on the unemployment effect of various environmental policies (Wagner, 2005; Hafstead and Williams, 2018).

Aubert and Chiroleu-Assouline (2017) analytically examines the efficiency consequences of an emission tax in a search equilibrium framework. Their paper differs from our model in several aspects. First, their paper does not model polluting and nonpolluting sectors explicitly, shutting down the important channel of sectoral reallocation. Second, their model considers heterogeneous agents (i.e., high-skill and low-skill workers), with full employment for high-skill workers. Third, it abstracts an abatement decision (i.e., the intensive margin of emission) from the model, which is a crucial piece of our analysis. Fourth, their paper is silent on intensity standards.

In addition to the above analytical works, this paper also complements the numerical studies in the literature. This paper and two recent studies (Hafstead et al., 2018; Hafstead and Williams, 2018) combine the strengths of analytical and numerical approaches: while our stylized analytical model uncovers major mechanisms at play, their numerical models measure the empirical significance of these mechanisms. Hafstead et al. (2018) compare the labor market outcomes of a revenue-neutral emission tax between a full-employment computable-general equilibrium (CGE) model and a search-CGE model. Their paper is silent on intensity standards. Hafstead and Williams (2018) evaluate the unemployment effects and the welfare costs of intensity standards and revenue-neutral emission taxes in a general equilibrium two-sector search model. In contrast to particular policy scenarios in their studies, this paper generalizes the theory to broader policy scenarios, uncovers the key determinants of the tradeoff between environmental quality and employment opportunity, characterizes the optimal environmental policies in the imperfect labor market, and uncovers the welfare tradeoff from various dimensions: the production externalities along both margins and the job allocation externality.

The rest of the paper is organized as follows. Section 2 presents the structure of the model. Section 3 derives the sufficient statistics that allow us to draw a conclusion on the inevitable tradeoff between employment and environmental quality among different types of policies in the search equilibrium framework. Section 4 discusses the tradeoff of environmental policies between environment and employment. Section 5 performs a conventional welfare analysis of the considered policies. Section 6 concludes.
2 The Basic Model

This section constructs a simple search equilibrium model featuring (i) involuntary unemploy-
ment and (ii) extensive and intensive margins of emission, all of which are endoge-
nized. The model provides major mechanisms through which policies affect the tradeoff
between employment (i.e., frictional unemployment) and environmental quality (the emis-
sions along the two margins).

Time is continuous. There is a continuum of utility-maximizing workers and profit-
maximizing job positions. Both workers and job positions are risk-neutral and discount
the future with an identical real interest rate $r$. Workers are identical and live infinitely.
Without loss of generality, we normalize the measure of workers to unity.

2.1 Technology

To produce the final good, the economy first uses labor and capital to produce two non-
storable intermediate goods, sells them in a competitive market, and then immediately
transforms them into the final consumption good. The technology of production for the
final good is

$$Y = \left( \alpha Y_d^\rho + (1 - \alpha) Y_c^\rho \right)^{\frac{1}{\rho}},$$

where $Y_d$ is the aggregate production of the first input, $Y_c$ is the aggregate production of the
second input, $\rho < 1$, and $\alpha \in (0, 1)$. The production of $Y_d$ will emit harmful substances
that pollute the environment, and the production of $Y_c$ does not pollute the environment.$^4$
The subscript $d$ denotes “dirty”, and the subscript $c$ denotes “clean”. We call $Y_d$ the dirty
good and $Y_c$ the clean good. The elasticity of substitution between the dirty good and the
(clean good is $1/(1 - \rho)$. $\alpha$ is a parameter for the relative importance of the dirty good.
Equation (1) allows us to interpret the final consumption good as a utility index defined
over the two intermediate goods. Throughout the paper, we will normalize the price of the

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$^3$This assumption is common in the literature (Mortensen and Pissarides, 1994; Moen, 1997; Moscarini, 2005;
Rogerson et al., 2005; Gonzalez and Shi, 2010; Fujita and Ramey, 2012; Michaillat, 2012). Applications of the
search and matching model also assume agents to be risk-averse, such as the literature that investigates the optimal
unemployment benefits with search frictions (Fredriksson and Holmlund, 2006; Guerrieri et al., 2010). Recent
literature also investigates job search behaviors with the preference of ambiguity aversion (Chan and Yip, 2019).
Our model can be easily extended to incorporate agents’ ambiguity preferences; we do not do so because the
modification of agents’ preference towards ambiguity complicates our model without providing a richer economic
intuition in our context.

$^4$This simplification is adopted in the search and matching literature (Acemoglu, 2001; Chassamboulli and
Palivos, 2014). A similar constant elasticity of substitution (CES) aggregation function is also used in Hafstead
and Williams (2018).
final good to unity.

Since the markets for the two intermediate goods are competitive, the prices of the intermediate goods are

\[ p_d = \alpha \left( \frac{Y}{Y_d} \right)^{1-\rho} \text{ and } p_c = (1 - \alpha) \left( \frac{Y}{Y_c} \right)^{1-\rho}. \] (2)

The technology of production for the intermediate goods is Leontief. When matched with a job position with the necessary equipment, a worker produces \( A \) units of the respective good. \( A > 0 \) is a parameter that measures productivity. The necessary equipment is sector-specific: a vacancy for producing intermediate good \( j \), once it is created, cannot be used to produce the other intermediate good.

### 2.2 Search and Match in the Labor Market

We assume that workers are either employed or unemployed. Only unemployed workers search. Job positions can choose to create either a vacancy for producing the dirty good or a vacancy for producing the clean good. The capital cost \( k \) is incurred before the job position creates a vacancy. A position must decide which good to produce before it creates a vacancy. There is free entry into both dirty-good and clean-good vacancies. At any point in time, a worker can work on only one job, and a vacancy can only be filled by one worker.

Search is undirected. Thus, both types of vacancies have the same probability of meeting workers. Workers and jobs come together via a matching technology \( M(u, v) \) where \( u \) is the unemployment rate, and \( v \) is the vacancy rate (the total number of vacancies).\(^5\) We make the standard assumptions in the literature: \( M(u, v) \) is twice differentiable, increasing in both arguments, has constant returns to scale, and satisfies the standard Inada-type assumptions. Thus, the flow rate of match for a vacancy is \( M(u, v)/v \equiv q(\theta) \), where \( q(\cdot) \) is a differentiable decreasing function, and \( \theta \equiv v/u \) is the tightness of the labor market. Thus, the flow rate of match for an unemployed worker is \( M(u, v)/u = \theta q(\theta) \).\(^6\) \( \theta q(\theta) \) is increasing in \( \theta \), \( \lim_{\theta \to 0} q(\theta) = \infty \), \( \lim_{\theta \to \infty} q(\theta) = 0 \), \( \lim_{\theta \to 0} \theta q(\theta) = 0 \), and \( \lim_{\theta \to \infty} \theta q(\theta) = \infty \). Finally, all filled positions end with the exogenous flow rate \( \lambda > 0 \).

Upon matching, the worker and the position bargain on the wage for producing good \( j \). After bargaining, production starts, the position sells the product and pays the wage to the worker.\(^7\) The worker holds the job until the separation shock arrives. When the separation

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\(^5\)Hafstead and Williams (2018) also assume that the two sectors hire from the same pool of workers.

\(^6\)Endogenizing search intensity in this model is simple but would not result in richer economic intuition. No results in this paper would be altered by endogenizing search intensity.

\(^7\)The model assumes that workers are hired only for production and not for other activities such as recruitment. Readers who are interested in theoretical frameworks in which workers engage in recruitment are referred to
shock arrives, the worker becomes unemployed and goes back to search. The flow return from unemployment is \( z \), where \( z \) can be interpreted as the level of utility derived from leisure or the value of home production.

### 2.3 Bellman Equations

Let \( J_j^E \) denote the value of employment on the job that produces good \( j \) and \( J_j^U \) the value of unemployment. \( w_j \) is the wage for producing good \( j \). \( J_j^E \) can be written as

\[
rJ_j^E = w_j + \lambda (J_j^U - J_j^E).
\]

Denote the proportion of dirty-good vacancies among all vacancies as \( \phi \). \( J_j^U \) can be written as

\[
rJ_j^U = z + \theta q(\theta) \left[ \phi(J_d^E - J^U) + (1 - \phi)(J_c^E - J^U) \right].
\]

Since both types of vacancies meet workers at the same rate in steady state, and workers accept both types of jobs in equilibrium, \( \phi \) is also the share of dirty-good jobs among all filled jobs.

Let \( J_d^F \) denote the value of a filled job that produces good \( j \), and \( J_c^V \) the value of a vacancy for producing good \( j \). Let \( x \) denote the level of pollutant emission of each filled job producing the dirty good. Let \( C(\bar{x} - x) \) denote the total cost of abating emission from \( \bar{x} \) (the unabated level) to \( x \) (the abated level). Jobs producing dirty goods choose the emission level \( x \) to maximize the asset value of a filled job. Hence, the asset value of a filled job producing good \( j \) is:

\[
rJ_d^F = \max_x \left[ p_d A - w_d - C(\bar{x} - x) + \lambda (J_d^V - J_d^F) \right]
\]
\[
rJ_c^F = p_c A - w_c + \lambda (J_c^V - J_c^F).
\]

We make the following assumptions on the cost function of abatement: \( C : [0, \bar{x}] \rightarrow \mathbb{R}_+ \) is twice differentiable, \( C(0) = 0 \), \( C'(\cdot) > 0 \), \( C''(\cdot) > 0 \), \( \lim_{x \to \bar{x}} C'(\bar{x} - x) = 0 \), and \( \lim_{x \to 0} C'(\bar{x} - x) = \infty \). So, the greater is abatement level, the costlier is an additional unit of abatement. The optimal level of emission per dirty-good job \( x \) satisfies the following first-order condition:

\[
C'(\bar{x} - x) = 0.
\]

Hafstead and Williams (2018). Also, the key results of this paper remain unchanged regardless of the presence of payroll tax.
Given the assumptions on the cost function of abatement, the optimal emission level \( x \) is given by \( \bar{x} \). While abatement is costly, it does not provide any economic benefit to dirty-good producers. As a result, they do not abate emission, and the abatement cost is \( C(\bar{x} - x) = 0 \).

The asset value of a vacancy for producing good \( j \) is:

\[
r_J^V = q(\theta) \left( J_j^F - J_j^V \right).
\]

Due to search friction, at the moment a worker finds a job, there is a bilateral monopoly, and this leads to rent sharing over the surplus of the match. We assume that the rent sharing rule is:

\[
(1 - \beta)(J_j^E - J_j^U) = \beta(J_j^F - J_j^V),
\]

where \( \beta \in (0, 1) \). This rule is the implication of Nash bargaining between risk-neutral workers and jobs who have the same discount rate, where the worker has bargaining power \( \beta \) (Pissarides, 2000).

The free entry and exit of vacancies drive the expected gross profits of vacancies to the machinery cost \( k \); thus

\[
J_j^V = k.
\]

Finally, in steady state, flows out of unemployment are equal to the number of separations from jobs. Thus

\[
u = \frac{\lambda}{\lambda + \theta q(\theta)}.
\]

### 2.4 Steady-State Equilibrium

**Definition 1.** A steady-state equilibrium is defined as \( \{Y, p_j, w_j, x, \phi, \theta, J_j^E, J_j^U, J_j^F, J_j^V\} \) such that for all \( j \in \{c, d\} \),

1. (Production of Final Goods): \( Y \) satisfies equation (1);
2. (Goods Market Clearing): Prices of two intermediate goods \( p_j \) satisfy equation (2);
3. (Value Functions): \( J_j^E, J_j^U, J_j^F, J_j^V \) satisfy equations (3), (4), (5), and (7);
4. (Optimal Emission Level): Emission level \( x \) satisfies equation (6);
5. (Rent Sharing): Wages for the two jobs \( w_j \) satisfy the sharing rule (8);
6. (Free Entry and Exit): The proportion of dirty-good jobs \( \phi \) and market tightness \( \theta \) satisfy equation (9);

\[8\text{ Chan and Yip (2019) shows that this rule is also implied if workers and jobs are ambiguity-averse.}\]
7. (Steady-State Accounting): The unemployment rate \( u \) satisfies equation (10).

We characterize the equilibrium objects as functions of \( \phi \) and \( \theta \). In a steady-state equilibrium, \( Y_d = (1 - u)\phi A \) and \( Y_c = (1 - u)(1 - \phi)A \). Using equation (2), the prices are

\[
p_d(\phi) = \alpha \left\{ \frac{[\alpha \phi^\rho + (1 - \alpha)(1 - \phi)^{\rho}]^\frac{1}{\rho}}{\phi} \right\}^{1 - \rho} \quad \text{and} \quad p_c(\phi) = (1 - \alpha) \left\{ \frac{[\alpha \phi^\rho + (1 - \alpha)(1 - \phi)^{\rho}]^\frac{1}{\rho}}{1 - \phi} \right\}^{1 - \rho}.
\]

\[
(11)
\]

\( p_d \) strictly decreases with \( \phi \), and \( p_c \) strictly increases with \( \phi \). In a steady-state equilibrium, \( \phi = Y_d/(Y_d + Y_c) \) is the quantity share of dirty goods in the market. The increase in \( \phi \) leads to an increase in the relativity quantity of the dirty goods, thus lowering its value.

From equations (3), (5), (8), and (9), the wage equations are

\[
w_d = (1 - \beta)rJ^U + \beta \left( p_d A - C(\bar{x} - x) - rk \right) \quad \text{and} \quad w_c = (1 - \beta)rJ^U + \beta (p_c A - rk).
\]

\[
(12)
\]

A worker is compensated with fractions of his outside option value and the flow profit in the corresponding sector. Using equations (4), (7), (8), and (9), we have

\[
rJ^U = z + \frac{\theta}{1 - \beta} rk.
\]

\[
(13)
\]

\( rJ^U \) strictly increases with both \( \theta \) and \( k \). Intuitively, higher market tightness shortens unemployment spell, increasing a worker’s outside option value.

Equations (7), (8), (12), and (13) imply the following zero-profit conditions:

\[
rk = q(\theta)(1 - \beta) \left( \frac{p_d(\phi) A - C(\bar{x} - x) - rk - z - \theta \beta rk}{1 - \beta} \right) \quad \text{and} \quad (14)
\]

\[
rk = q(\theta)(1 - \beta) \left( \frac{p_c(\phi) A - rk - z - \theta \beta rk}{1 - \beta} \right).
\]

\[
(15)
\]

Along each locus, vacancies in the corresponding sector make zero expected profits. Using the two zero-profits conditions (14) and (15), the revenue differential between the two jobs is

\[
\Lambda(\phi) \equiv p_d(\phi) A - p_c(\phi) A = 0,
\]

\[
(16)
\]

which implies that \( p_d = p_c \) in the equilibrium. Also, one can show that \( \Lambda'(\phi) < 0 \).
Proposition 1. There exists a unique steady-state equilibrium, as defined in Definition 1.

Proof. See Appendix 7.1.

Figure 1 provides a graphical presentation of Proposition 1. It illustrates the determination of the equilibrium. In the \( \phi - \theta \) plane, locus (14) slopes downward, and locus (15) slopes upward. An increase in the share of dirty-good jobs reduces the relative price of dirty goods, lowering the expected profits of its vacancies. Consequently, the supply of the dirty-good vacancies and thus market tightness decline. A similar intuition can be derived from the positive slope of the locus (15). The two loci intersect only once in the domain \( \phi \in (0, 1) \), and the equilibrium is the intersection of the two loci.

3 The Tradeoff Between Employment and Environment

In the steady-state equilibrium, there are \( \phi(1 - u) \) number of jobs producing the dirty good, and each job emits \( x \) units of pollutant. Denote the aggregate emission level (i.e., the steady-state level of emission stock) by \( \chi \), which is

\[
\chi \equiv x\phi(1 - u). \tag{17}
\]
There are three margins of change in the aggregate emission level, as the following equation shows:

\[
d\chi = \phi (1 - u) dx + x (1 - u) d\phi + x \phi d(1 - u).
\]

Intensive Margin  Extensive Margin  Scale Effect

Changes in \(x\) and \(\phi\) are adjustments in the intensive and the extensive margins of emission, respectively. A change in \(1 - u\) represents the scale effect on emission: holding emission per job and the composition of jobs constant, more output leads to more emission stock in a steady-state equilibrium. To normalize the output scale so that we can focus on the intensive and extensive margins of emission, we use an emission per unit of output (i.e., relative emission level) as a measure of environmental quality, which is

\[
\varphi \equiv \frac{\chi}{Y}.
\]

(18)

Using the above definition, equations (1) and (18) imply that

\[
\varphi = \frac{x \phi (1 - u)}{\left(\alpha Y_d^\rho + (1 - \alpha) Y_c^\rho\right)^{\frac{1}{\rho}}} = \frac{x \phi}{A \left(\alpha \phi^\rho + (1 - \alpha)(1 - \phi)^\rho\right)^{\frac{1}{\rho}}},
\]

(19)

which shows that \(\varphi\) strictly increases with \(x\) and \(\phi\). That is, emission (relative to output) increases, thus environmental quality deteriorates, along both the intensive and extensive margins.

In what follows, we formulate a general situation in which there is an additional flow cost \(T \geq 0\) imposed on dirty-good production by the government. Accordingly, we rewrite the zero-profit condition of a dirty-good job as

\[
rk = \frac{q(\theta)(1 - \beta)}{r + \lambda} \left( p_d(\phi) A - T - rk - z - \frac{\theta \beta rk}{1 - \beta}\right).
\]

(20)

The zero-profit condition of a clean-good job remains unchanged and is given by equation (15). The characteristic of this type of policy is that it only affects the flow cost of dirty-good production.

Because unemployment strictly decreases with \(\theta\) according to equation (10), \(u\) tends to zero when \(\theta\) approaches positive infinity and to one when \(\theta\) approaches zero, we can map the equilibrium conditions (14) and (15) from the \(\phi-\theta\) plane to the \(\phi-u\) plane as shown in Figure 2. Clearly, if locus (14) shifts up along locus (15) while locus (15) is fixed, unemployment \(u\) will go up while \(\phi\) goes down; that is, a tradeoff between employment

9The policy instrument, intensity standard, shares a similar notion of this criteria. While intensity standards limit emissions per unit of dirty-good output, \(\varphi\) uses emissions per unit of final-good output to compare environmental quality among environmental policies.
and environmental quality along its extensive margin is inevitable. It is straightforward to show that the larger the cost $T$, the larger the unemployment rate, and the smaller the share of polluting sector. This can be seen in Figure 2 that the larger the cost $T$, the more the locus (20) shifts upward. Consequently, the unemployment rate will increase more, and the share of dirty-good jobs will decrease more. Such impacts can also be seen from the revenue differentials between the two jobs:

$$\Lambda(\phi) = T.$$  

(21)

A higher $T$ implies a lower $\phi$ in a steady-state equilibrium because $\Lambda'(\phi) < 0$. Intuitively, a higher flow cost makes the production of dirty goods costlier, reducing the supply of its vacancies $\theta$ and its market share $\phi$. Since clean goods become more abundant, its price goes down, depressing the supply of its vacancies. This general equilibrium effect further decreases market tightness $\theta$, and thus increases $u$.

We define by $P(x, T)$ a policy that increases the flow cost by $T$ in the polluting sector with the intensive margin of emission $x$. The following Theorem summarizes the tradeoff between employment and environment.

**Proposition 2.** If $x_1 \leq x_2$ and $T_1 > T_2$, $P_1(x_1, T_1)$ and $P_2(x_2, T_2)$ cannot achieve a better environmental quality and a lower unemployment rate at the same time than each other.

Notice that $P(\bar{x}, 0)$ is a “policy” or a situation which incurs no cost in the two sectors.
and does not incent firms to abate emission. If \( x \leq \bar{x} \) and \( T > 0 \), the tradeoff between employment and environment is inevitable between \( P(x, T) \) and \( P(\bar{x}, 0) \). Since the intensive margin of emission is always no less than \( \bar{x} \), the proposition concludes that no policy that incurs a higher flow cost in the polluting sector can improve environmental quality without creating involuntary unemployment. This class of policy includes a lump-sum tax on the polluting sector, simple emission tax with the emission tax revenue abstracted from the labor market, intensity standards, etc.

This proposition lays the theoretical foundation of the conflicts between two lobbying groups in the literature on the political economy of environmental policy (Aidt, 1998; Fredriksson and Svensson, 2003; Oates and Portney, 2003; List and Sturm, 2006). Local environmental organizations lobby against the situation \( P(\bar{x}, 0) \) because there always exists a broad class of policies \( P(x, T) \) that could improve environmental quality. Polluting industries and labor unions lobby against these broad classes of policies because any proposed environmental policies in this class must scarify job opportunities.

This proposition proves a theory in understanding the economic impacts of environmental regulation policies: regulation is costly. In the simplest case where the regulation only affects the economic parameters of the polluting sector, it is clear why regulating dirty-good production will cost jobs. The reason is that the increase in the flow cost of dirty-good vacancies depreciates their value. Firms will then open less dirty-good vacancies relative to clean-good vacancies. The change in job composition translates into changes in relative prices and in turn affects the profit conditions of both types of job. The value of clean-good vacancies will also be negatively affected because of the downward general equilibrium effect on clean-good price. The outcome is that the number of vacancies will decrease and the market will become tighter. Consequently, unemployment rises.

Meanwhile, a policy with higher flow cost in the polluting sector improves emission along its extensive margin. Despite there being no direct flow cost in clean-good jobs, this proposition does allow the general equilibrium effect of the flow cost in dirty-good jobs on clean-good jobs. The primary effect of the direct cost \( T \) dominates and reduces the market share of the polluting sector, the extensive margin of emission. The policy reduces emission even though it may not reduce emission along its intensive margin.

Proposition 2 can also be utilized to compare policies with different combinations of \( x \) and \( T \). In the next section, we will demonstrate the application of Proposition 2 to compare simple emission tax and intensity standard.
4 Comparisons Between Environmental Policies

4.1 Comparisons Between Simple Emission Tax and Intensity Standard

Denote a simple emission tax policy of a rate $\tau > 0$ and an intensity standard that caps the level of emission per $A$ unit of dirty good at $\hat{x} \in (0, \bar{x})$ by $ET(\tau)$ and $IS(\hat{x})$.

Under $ET(\tau)$, firms that produce dirty goods choose the emission level $x$ to maximize the asset value of a filled job. Thus, the asset value of each job is given by:

$$r_J^F = \max_{x \in [0, \bar{x}]} p_dA - w_d - C(\bar{x} - x) - \tau x + \lambda(J_d^V - J_d^F) \text{ and}$$

$$r_J^F = p_cA - w_c + \lambda(J_c^V - J_c^F).$$

(22)

The optimal emission level for each dirty-good job $x_{ET}(\tau)$ is the solution to the following first-order condition:

$$C'(\bar{x} - x_{ET}(\tau)) = \tau,$$

(23)

Intuitively, an additional unit of abatement increases abatement cost by the marginal abatement cost but saves emission tax payment of this additional unit. In other words, the optimal emission level equates the marginal abatement cost to its marginal abatement benefit. Under the assumptions $\lim_{x \to \bar{x}} C'(\bar{x} - x) = 0$ and $\lim_{x \to 0} C'(\bar{x} - x) = \infty$, the intermediate value theorem ensures the existence of $x \in (0, \bar{x})$ that solves equation (23). Since $C''(\bar{x} - x) > 0$, the optimal emission level $x_{ET}(\tau)$ is unique. The flow cost of $ET(\tau)$ is $T_{ET}(\tau) = C(\bar{x} - x_{ET}(\tau)) + \tau x_{ET}(\tau)$.

Under $IS(\hat{x})$, the asset value of each job is given by:

$$r_J^F = \max_{x \in [0, \hat{x}]} p_dA - w_d - C(\bar{x} - x) + \lambda(J_d^V - J_d^F) \text{ and}$$

$$r_J^F = p_cA - w_c + \lambda(J_c^V - J_c^F).$$

(24)

Since $C(\bar{x} - \hat{x})$ is strictly increasing, the optimal abatement level equals $\bar{x} - \hat{x}$. The flow cost of $IS(\hat{x})$ is $T_{IS}(\hat{x}) = C(\bar{x} - \hat{x})$.

The tradeoff between employment and environment is inevitable between simple emission tax $ET(\tau)$ and the intensity standard $IS(x_{ET}(\tau))$ with an implicit emission tax rate equal to the simple emission tax policy. With the implicit emission tax rate, the optimal emission levels are identical, and thus abatement costs are the same (i.e., $C(\bar{x} - x_{ET}(\tau))$). In addition to the abatement cost, the polluting firms are required to pay an emission tax under $ET(\tau)$ but not under $IS(x_{ET}(\tau))$. Consequently, the flow cost is higher under $ET(\tau)$ than under $IS(x_{ET}(\tau))$ (i.e., $T_{ET}(\tau) > T_{IS}(x_{ET}(\tau))$). Hence, $ET(\tau)$ results in a higher
unemployment rate but a lower emission level along an extensive margin than \( IS(x^{ET}(\tau)) \). With the identical intensive margin of emission but the lower level along the extensive margin, \( ET(\tau) \) can achieve a better environmental quality than the equivalent \( IS(x^{ET}(\tau)) \).

The direct application of Proposition 2 leads to the following corollary.

**Corollary 1.** For any emission tax rate \( \tau > 0 \), \( ET(\tau) \) and \( IS(x^{ET}(\tau)) \) cannot achieve a better environmental quality and a lower unemployment rate at the same time than each other.

Corollary 1 implies that simple emission tax and the equivalent intensity standard do not dominate each other. This conclusion is made simply using two sufficient statistics—the intensive margin of emission and flow cost. While unemployment (or employment) is a function of market tightness \( \theta \) (see equation (10)), the environmental quality depends on the market share of the polluting sector \( \phi \) (see equation (19)). Both the market tightness and market share, as illustrated in Figure 1, jointly characterize the equilibrium. Proposition 2 allows us to compare the unemployment and environmental quality between any two policies without solving the entire structural model. Moreover, the two sufficient statistics can be easily obtained from the firm problem—a partial equilibrium. For example, once the optimal emission level under \( ET(\tau) \) is pinned down by equation (23), the computation of the corresponding flow cost is straightforward. The conclusion of the inevitable tradeoff follows.

Next, we ask whether there exists any intensity standard that dominates simple emission tax in terms of employment and environment? In other words, given any emission tax rate \( \tau > 0 \), does it exist any standard \( \hat{x} \) such that \( IS(\hat{x}) \) can achieve better environmental quality and a lower unemployment rate than \( ET(\tau) \)? Similarly, given any \( \hat{x} \), does it exist any tax rate \( \tau \) such that \( ET(\tau) \) dominates \( IS(\hat{x}) \)?

These two questions are important because the two environmental policies are commonly seen in practice. To the extent that policymakers aim to improve the environmental quality while maintaining the unemployment rate, the answers to these questions are highly relevant to environmental lawmaking. Suppose there is a prevalent emission tax of a rate \( \tau \) in the economy, and the policymaker considers replacing the tax by intensity standard. The literature does not adequately answer whether there exist any intensity standards such that this policy shift simultaneously reduces unemployment and improves the environmental quality or, similarly, whether there exist any emission taxes \( \tau \) such that the policy shift from intensity standard to emission tax is preferred.

Before answering the two questions, it is a good time to explore the properties of simple emission tax and intensity standard. First, it is straightforward to see from equation (23) that the optimal emission level \( x^{ET}(\tau) \) strictly decreases with \( \tau \). Second, the flow cost is equal to the sum of abatement cost and emission tax payment under \( ET(\tau) \). That is, \( T^{ET}(\tau) = C(\bar{x} - x^{ET}(\tau)) + \tau x^{ET}(\tau) \). By the envelope theorem, the flow cost strictly increases with an emission tax rate. As illustrated in Figure 2, a higher flow cost results in
a higher unemployment rate and a smaller market share of the polluting sector. With lower emission levels along the extensive and the intensive margins, a higher emission tax rate results in a better environmental quality. The following lemma summarizes the results:

**Lemma 1.** For all \( \tau \in \mathbb{R}_+ \),

\[
\begin{align*}
\frac{dx^{ET}(\tau)}{d\tau} < 0, \\
\frac{dT^{ET}(\tau)}{d\tau} > 0, \\
\frac{d\phi^{ET}(\tau)}{d\tau} < 0, \\
\frac{d\varphi^{ET}(\tau)}{d\tau} < 0, \text{ and} \\
\frac{du^{ET}(\tau)}{d\tau} > 0.
\end{align*}
\]

Second, the flow cost strictly decreases with the standard \( \hat{x} \) under \( IS(\hat{x}) \). The reason is that a lower \( \hat{x} \) requires a higher level of abatement and incurs a higher abatement cost. Hence, Lemma 2 follows:

**Lemma 2.** For all \( \hat{x} \in (0, \bar{x}) \),

\[
\begin{align*}
\frac{dx^{IS}(\hat{x})}{dx} > 0, \\
\frac{dT^{IS}(\hat{x})}{dx} < 0, \\
\frac{d\phi^{IS}(\hat{x})}{dx} > 0, \\
\frac{d\varphi^{IS}(\hat{x})}{dx} > 0, \text{ and} \\
\frac{du^{IS}(\hat{x})}{dx} > 0.
\end{align*}
\]

We are ready to answer the two questions. First, we assume the economy begins with a prevalent emission tax with a rate \( \tau > 0 \). Given \( \tau \), the policy \( ET(\tau) \) incurs a flow cost \( T^{ET}(\tau) > 0 \). When \( \hat{x} \) equals \( \bar{x} \), no abatement is required and thus \( T^{IS}(\bar{x}) = 0 \). When \( \hat{x} \) approaches zero, no emission is allowed and thus the flow cost approaches infinity. The intermediate value theorem ensures the existence of a standard \( x_T(\tau) \) such that the flow costs are identical under the two policies (i.e., \( T^{IS}(x_T(\tau)) = T^{ET}(\tau) \)). By Lemma 2, \( dT^{IS}(\hat{x})/\hat{x} < 0 \) ensures that the standard \( x_T(\tau) \) is unique. The corresponding market share is denoted by \( \phi^{IS}(x_T(\tau)) \).

For all \( \hat{x} \leq [0, x_T(\tau)] \), intensity standard \( IS(\hat{x}) \) leads to a higher unemployment rate than the prevalent policy \( ET(\tau) \). When \( \hat{x} = x_T(\tau) \), the flow costs, the unemployment rates, and the market shares are identical under the two policies, as shown at point A in Figure 3. By Lemma 2, the higher the \( \hat{x} \), the lower the unemployment rate. Hence, there exists

![Figure 3: Comparisons between emission tax and intensity standard](image-url)
no standard \( \hat{x} \leq [0, x_T(\tau)) \) that will lead to a lower unemployment rate than the prevalent policy. In other words, the unemployment rate is higher under IS(\( \hat{x} \)) than under ET(\( \tau \)) for all \( \phi < \phi^{IS}(x_T(\tau)) \). A similar argument can be applied to show that unemployment under IS(\( \hat{x} \)) is no more than that under ET(\( \tau \)) for all \( \hat{x} \leq [x_T(\tau), \bar{x}] \) (or for all \( \phi \in [\phi^{IS}(x_T(\tau)), \phi^{IS}(\bar{x})] \)).

However, the environmental quality is lower under intensity standard IS(\( \hat{x} \)) than under the prevalent emission tax policy for all \( \hat{x} \leq [x^{ET}(\tau), \bar{x}] \). As mentioned above, the flow cost is higher under the prevalent emission tax policy than the corresponding equivalent intensity standard IS(\( x^{ET}(\tau) \)) because no emission tax payment is incurred under intensity standard. With a lower flow cost, the extensive margin of emission is larger under the equivalent intensity standard. By definition, the intensive margins of emission are identical under the two policies. Hence, the environmental quality is lower under the equivalent intensity standard IS(\( x^{ET}(\tau) \)) than under the prevalent emission tax policy. Therefore, the intensity standard IS(\( x^{ET}(\tau) \)) cannot achieve better environmental quality than the prevalent policy, as shown at point C in Figure 3. By Lemma 2, the higher the \( \hat{x} \), the poorer the environmental quality. We can, therefore, conclude that there exists no intensity standard \( \hat{x} \in [x^{ET}(\tau), \bar{x}] \) that will lead to a better environmental quality than the prevalent policy. Correspondingly, intensity standard with the market share \( \phi \in [\phi^{IS}(x^{ET}(\tau)), \phi^{IS}(\bar{x})] \) results in a worse environmental quality than the prevalent policy.

To summarize the above analyses, intensity standard with the market share to the left of \( \phi^{IS}(x_T(\tau)) \) does not dominate the prevalent policy because of the higher unemployment rate. The standard with the market share to the right of \( \phi^{IS}(x^{ET}(\tau)) \) does not dominate because of the poor environmental quality. In the rest of analyses, we will show that there exists an interval of intensity standard that can achieve better environmental quality and a lower unemployment rate.

Notice that the flow costs are identical under IS(\( x_T(\tau) \)) and the prevalent policy. Therefore, the extensive margins of emission are identical under the two policies. Meanwhile, IS(\( x_T(\tau) \)) incurs a higher abatement cost. In other words, the emission level along the intensive margin is lower under IS(\( x_T(\tau) \)). That is, \( x_T(\tau) < x^{ET}(\tau) \). Hence, the environmental quality is better under IS(\( x_T(\tau) \)) than the prevalent emission tax policy.

While the intensity standard gives a better environmental quality than the prevalent emission tax policy at \( \phi^{IS}(x_T(\tau)) \), its environmental quality is lower than that under the prevalent policy at \( \phi^{IS}(x^{ET}(\tau)) \). By Lemma 2, there must exist a unique standard \( x_c(\tau) \) such that the environmental qualities are identical under IS(\( x_c(\tau) \)) and ET(\( \tau \)), as shown at point B. By Lemma 2, the higher the \( \hat{x} \), the poorer the environmental quality. For all \( \hat{x} \in [x_T(\tau), x_c(\tau)] \) (or between \( \phi^{IS}(x_T(\tau)) \) and \( \phi^{IS}(x_c(\tau)) \), IS(\( \hat{x} \)) results in a better environmental quality than ET(\( \tau \)). Proposition 3 summarizes the results.

**Proposition 3.** Given any emission tax \( \tau \in \mathbb{R}_{++} \), there always exists a corresponding interval of intensity standard \( \hat{x} \in (x_T(\tau), x_c(\tau)] \) such that IS(\( \hat{x} \)) improves environment-
tal quality and lowers unemployment, where $x_T(\tau)$ solves $T^{IS}(x_T(\tau)) = T^{ET}(\tau)$, and $x_e(\tau) \in (x_T(\tau), x^{ET}(\tau))$.

In what follows, we answer whether there exists such an emission tax rate that dominates any intensity standard. Again, we assume the economy begins with a prevalent intensity standard $IS(\hat{x})$ with a binding standard $\hat{x} \in (0, \bar{x})$. Under this policy, the optimal emission level equals the standard, and the flow cost of each job position equals the abatement cost. That is, $x^{IS}(\hat{x}) = \hat{x}$ and $T^{IS}(\hat{x}) = C(\bar{x} - \hat{x}) > 0$. Notice that if an emission tax rate is zero, the policy will provide no incentive to abatement. Hence, the abatement cost and the emission tax payment are zero. Hence, the flow cost is zero under $ET(0)$ (i.e., $T^{ET}(0) = 0$). While an emission tax rate approaches infinity, it will be too expensive to emit pollutants. As a result, the optimal emission level approaches zero and the abatement cost approaches infinity. In this case, the flow cost approaches infinity. By the intermediate value theorem, there must exist an emission tax rate $\tau_T(\hat{x})$ such that the flow costs are identical under $ET(\tau_T(\hat{x}))$ and the prevalent policy $IS(\hat{x})$. That is, $T^{ET}(\tau_T(\hat{x})) = x^{IS}(\hat{x})$.

By Lemma 1, the flow cost strictly increases with an emission tax rate; thus, the $\tau_T(\hat{x})$ is unique.

For all tax rates $\tau > \tau_T(\hat{x})$, an emission tax policy leads to a higher unemployment rate than the prevalent policy. Since the flow costs are identical under $ET(\tau_T(\hat{x}))$ and the prevalent policy, the unemployment rates are identical, as shown in Figure 2. By Lemma 1, the unemployment rate strictly increases with an emission tax rate; hence, the unemployment rate under $ET(\tau)$ exceeds the rate under the prevalent policy for all $\tau > \tau_T(\hat{x})$.

The environmental quality is better under the prevalent policy than under an emission tax policy for all $\tau \in [0, \tau_T(\hat{x})]$. Since the flow costs are identical under the prevalent policy and the emission tax policy $ET(\tau_T(\hat{x}))$, their extensive margins of emission are the same. While the entire flow cost is used for abatement under the intensity standard, a portion of the flow cost is the emission tax payment under the emission tax. This explains why the emission level is lower under the prevalent policy than under $ET(\tau_T(\hat{x}))$ along the intensive margin. Hence, the prevalent policy could achieve better environmental quality than $ET(\tau_T(\hat{x}))$. By Lemma 1, the environmental quality is lower at lower emission tax rates. Hence, any emission tax policy with the tax rate $\tau \in [0, \tau_T(\hat{x})]$ results in the lower environmental quality than the prevalent policy.

Therefore, given any intensity standard, there does not exist any emission tax rate that could improve environmental policy without increasing unemployment. Proposition 4 summarizes the results.

**Proposition 4.** Given any intensity standard $\hat{x} \in (0, \bar{x})$, there exists no emission tax rate $\tau \in \mathbb{R}_+^+$ such that $ET(\tau)$ improves environmental policy without increasing unemployment.

This subsection concludes that an intensity standard is superior to a simple emission tax
in terms of environmental quality and unemployment. This result complements Hafstead and Williams (2018). Their paper shows that sectoral shifts between sectors are smaller under intensity standard. This subsection shows that for any emission tax rate, there always exists an interval of intensity standard that improves environmental quality and lowers unemployment. Conversely, no emission tax rate dominates any intensity standard. While taxes are generally first-best, Tombe and Winter (2015) summarizes the following factors that favor intensity standards: market power (Li and Shi, 2011), incomplete regulation or leakage (Holland, 2012), learning-by-doing in production (Gerlagh and Van der Zwaan, 2006), pre-existing tax distortions (Parry and Williams, 2012), unexpected productivity shocks (Fischer and Springborn, 2011). This subsection contributes to this literature by highlighting that the tradeoff between environment and employment is one of the factors favoring intensity standard over emission tax.

4.2 Revenue-Neutral Emission Tax

Next, we consider a revenue-neutral emission tax policy: an emission tax is levied on dirty-good producers, and the tax revenue is redistributed to the two sectors. While the “job-killing nature” of emission taxes and other environmental policies may have undermined environmental lawmaking, this subsection answers whether the rebate of the emission tax revenue could help simple emission tax avoid from the tradeoff between environmental quality and employment. Moreover, this section answers whether the dominating feature of intensity standard, shown in the previous subsection, remains intact if the emission tax revenue is rebated. While revenue-neutral emission tax are the topic of many recent studies (Goulder, 1995; Parry, 1995; Parry and Bento, 2000; Chiroleu-Assouline and Fodha, 2006; Bento and Jacobsen, 2007; Dissou and Sun, 2013; Hafstead and Williams, 2018; Yip, 2018), our analytical results are the first to compare this policy to other environmental policies using in terms of environmental quality and unemployment.

Denote $\mathcal{R}N(\tau, h)$ as a revenue-neutral emission taxation scheme with the tax revenue redistributed to subsidize each filled job in sector $j$ by $h_j$ dollars in a lump-sum fashion, $h \equiv \{h_c, h_d\}$. Since this policy is revenue-neutral, $h_c$ and $h_d$ satisfy the following budget constraint:

$$\phi^{\mathcal{R}N}(\tau, h)\tau x^{\mathcal{R}N}(\tau, h) = (1 - \phi^{\mathcal{R}N}(\tau, h))h_c + \phi^{\mathcal{R}N}(\tau, h)h_d.$$  \hspace{1cm} (25)

Taking $h_j$ as given, the asset values of filled jobs are

$$rJ_d^F = \max_{x \in [0, \bar{x}]} \left\{ p_d A - w_d - C(\bar{x} - x) - \tau + h_d + \lambda(J_d^V - J_d^F) \right\} \text{ and}$$

$$rJ_c^F = p_c A - w_c + h_c + \lambda(J_c^V - J_c^F).$$ \hspace{1cm} (26)
Using equations (7), (9), (26), and (27), the two zero-profit conditions are given by

$$h \frac{\partial N}{\partial \tau} = 0, \quad \text{and} \quad \frac{\partial E}{\partial \tau} = 0.$$  

Since $h$ as given, the optimal emission level $x^{RN}(\tau, h)$ solves the first-order condition (23) (i.e., $x^{RN}(\tau, h) = x^{ET}(\tau)$). The wage equations are given by

$$w_d = (1 - \beta)rJ^U + \beta \left( p_d A - C(\bar{x} - x) - \tau x - rk + h_d \right)$$  
and

$$w_c = (1 - \beta)rJ^U + \beta \left( p_c A - rk + h_c \right). \quad (27)$$

Using equations (7), (9), (26), and (27), the two zero-profit conditions are given by

$$rk = \frac{q(\theta)(1 - \beta)}{r + \lambda} \left( p_d (\phi) A - C(\bar{x} - x) - \tau x - z - rk \left( 1 + \frac{\theta \beta}{1 - \beta} \right) + h_d \right) \quad \text{and}$$  

$$rk = \frac{q(\theta)(1 - \beta)}{r + \lambda} \left( p_c (\phi) A - z - rk \left( 1 + \frac{\theta \beta}{1 - \beta} \right) + h_c \right). \quad (28)$$

It is straightforward to show the existence of the unique steady-state equilibrium and thus the proof is omitted for space consideration.

With the same emission level from the intensive margin, the abatement cost and the emission tax payment are identical under $RN(\tau, h)$ and $ET(\tau)$. Hence, the revenue differential can be expressed as follows:

$$\Lambda(\phi^{RN}(\tau, h)) = \Lambda(\phi^{ET}(\tau)) + h_c - h_d, \quad (29)$$

where $h_c$ and $h_d$ satisfy the budget constraint (25).

The rebate could improve the environmental quality. If $h_c = h_d$, the rebate does not change the incentive to create vacancies in one sector relative to the other. In this case, both the intensive and the extensive margins of emission are identical under $RN(\tau, h)$ and $ET(\tau)$. There is no difference in the environmental quality between the two policies. If $h_c > h_d$, it is less beneficial to open up vacancies in the polluting sector under $RN(\tau, h)$ than $ET(\tau)$. As a result, $\phi^{RN}(\tau, h) < \phi^{ET}(\tau)$. The environmental quality is strictly better under $RN(\tau, h)$ than $ET(\tau)$ if $h_c > h_d$.

A direct comparison between the zero-profit conditions of clean-good jobs (15) and (28) suggests that market tightness is higher under $RN(\tau, h)$ than $ET(\tau)$ if $p_c(\phi^{RN}(\tau, h)) + h_c \geq p_c(\phi^{ET}(\tau))$. When $h_c \geq h_d$, $\phi^{RN}(\tau, h) \leq \phi^{ET}(\tau)$. So, $p_c^{RN}(\tau, h) \leq p_c^{ET}(\tau)$, and $p_d^{RN}(\tau, h) \geq p_d^{ET}(\tau)$. The comparison between the prices of clean goods cannot conclude anything about market tightness. The revenue differential (29) implies that

$$p_d(\phi^{RN}(\tau, h)) - p_d(\phi^{ET}(\tau)) + h_d = p_c(\phi^{RN}(\tau, h)) - p_c(\phi^{ET}(\tau)) + h_c.$$

Since $h_c > h_d$ implies $p_d^{RN}(\tau, h) > p_d^{ET}(\tau)$, $h_c > h_d \geq 0$ implies that both the LHS and the RHS are nonnegative in the above equation. Therefore, if $h_c > h_d \geq 0$, $u^{RN}(\tau, h) <$
In case of $h_c = h_d$, $p_c^{RN}(\tau, h) = p_c^{ET}(\tau)$ and the two subsidies $h_j$ are positive. Hence, $u^{RN}(\tau, h) < u^{ET}(\tau)$. The following proposition summarizes the result.

**Proposition 5.** Given any emission tax rate $\tau \in \mathbb{R}^{++}$, if $h_c > h_d \geq 0$, $RN(\tau, h)$ will lead to a better environmental quality and a lower unemployment rate than $ET(\tau)$. If $h_c = h_d$, $RN(\tau, h)$ will lead to the same environmental quality and a lower unemployment rate than $ET(\tau)$.

The condition $h_c \geq h_d$ ensures that it is more profitable to create vacancies in the nonpolluting sector, lowering the market share of dirty-good jobs and thus the extensive margin of emission. Notice that emission along its intensive margin is identical under the two policies. The environmental quality is better under $RN(\tau, h)$ than $ET(\tau)$. With the higher market share of the nonpolluting sector, the price of clean goods are lower under $RN(\tau, h)$ than $ET(\tau)$. The condition $h_d \geq 0$ restricts the revenue differential being so large that the reduction in the price of clean goods will bring down the flow profits in the nonpolluting sector even in the presence of subsidies $h_c$. In other words, $h_d \geq 0$ ensures it is more beneficial to create jobs under $RN(\tau, h)$ than $ET(\tau)$ so that the unemployment rate is lower under $RN(\tau, h)$.

While Lemma 1 implies that the introduction of $ET(\tau)$ improves environment at the expense of unemployment, Proposition 5 states that a policy shift from $ET(\tau)$ to $RN(\tau, h)$ improves environment and depresses unemployment under certain conditions. Lemma 1 and Proposition 5 are not sufficient to conclude whether there exists $RN(\tau, h)$ that could improve environment without increasing involuntary unemployment. The following Proposition answers this question.

**Proposition 6.** If $\varepsilon_{p_d,\phi} \leq \varepsilon_{p_c,1-\phi}$ in the absence of any environmental policy, there always exists an emission tax rate $\tau \in \mathbb{R}^{++}$ such that the introduction of $RN(\tau, h)$ improves environmental quality and lowers unemployment, where $h_c = h_d$, and $\varepsilon_{p_d,\phi} \equiv \phi \frac{\partial p_d}{\partial g}$ and $\varepsilon_{p_c,1-\phi} \equiv (1-\phi) \frac{\partial p_c}{\partial (1-\phi)}$ are the partial elasticities of revenue with respect to respective market share.

**Proof.** See Appendix 7.2. \qed

According to Lemma 1 and Proposition 5, if $h_c = h_d$, the introduction of $RN(\tau, h)$ reduces emission along both margins. We do not repeat the intuition for space consideration.

$RN(\tau, h)$ could reduce unemployment. Intuitively, the introduction of an emission tax under $RN(\tau, h)$ increases the flow cost of dirty-good jobs, depressing the market share of the polluting sector. As the clean goods become relatively more abundant, there is a downward general equilibrium effect on the price of clean goods. This general equilibrium effect depresses the supply of clean-good vacancies and thus increases the unemployment rate. On the other hand, the transfer to filled jobs induces the supply of vacancies in both sectors, causing the unemployment rate to fall. Therefore, whether $RN(\tau, h)$ depresses
unemployment is inconclusive, dependent on the elasticities of revenues with respect to the market share.

The condition stated in Proposition 6 ensures that the responsiveness of the revenue of clean goods with respect to the emission tax is low. As such, the decrease in the flow profit in clean-good jobs from the negative general equilibrium effect is so small that this negative effect is outweighed by the direct effect of the transfer on their flow profit. Hence, the stated condition in Proposition 6 ensures that the introduction of a revenue-neutral emission tax will lower unemployment so that $\mathcal{RN}(\tau, h)$ could improve environmental quality and lower unemployment.

The above results do not adequately answer whether $\mathcal{RN}(\tau, h)$ and $\mathcal{IS}(\hat{x})$ are dominant. While Propositions (3) and (4) indicate intensity standard dominates emission tax if abstracting the tax revenue from the labor market, Proposition (5) states that revenue-neutral emission tax is superior to simple emission tax. How do we compare revenue-neutral emission tax with intensity standard? Proposition 7 answers this question.

**Proposition 7.** Suppose $h_c = h_d$. If the introduction of $\mathcal{RN}(\tau, h)$ improves environment and lowers unemployment, the policy shift from $\mathcal{RN}(\tau, h)$ to intensity standard $\mathcal{IS}(\hat{x}(\tau))$ will worsen the environmental quality for all $\hat{x}(\tau) \in [x_c(\tau), \bar{x}]$ and will increase unemployment for all $\hat{x} \in \mathbb{R}_+$. 

Proposition 7 follows immediately from a mix of the lemmas and the propositions discussed above. According to the discussion on Proposition 3, the environmental quality is strictly better under simple emission tax than under intensity standard for all $\hat{x}(\tau) \in [x_c(\tau), \bar{x}]$. Proposition 5 indicates that if $h_c = h_d$, the environmental quality is identical under emission tax whether the tax revenue is rebated to firms or abstracted from the labor market. The two propositions imply that $\mathcal{RN}(\tau, h)$ leads to a better environmental quality than intensity standard for all $\hat{x}(\tau) \in [x_c(\tau), \bar{x}]$. Next, Lemma 2 suggests that the introduction of intensity standard worsens unemployment. If the introduction of $\mathcal{RN}(\tau, h)$ lowers unemployment, the unemployment rate must be lower under $\mathcal{RN}(\tau, h)$ than under any intensity standard.

This subsection highlights an important message about revenue-neutral emission tax. Under certain condition, there always exists a revenue-neutral emission tax that improves environmental quality and depresses unemployment. Moreover, this revenue-neutral emission tax is superior to all simple emission tax policies and a large family of intensity standard policies. While Propositions 3 and 4 indicate the dominating feature of intensity standard over simple emission tax, this subsection finds that revenue-neutral emission tax dominates a large family of intensity standard policies. The rebate of the emission tax revenue means a lot to the double-dividend hypothesis—emission tax reduces emission, and rebate mitigates the distortion of the pre-existing tax, potentially improving welfare. This subsection contributes to the literature on the double-dividend hypothesis: the rebate matters a lot.
in avoiding the tradeoff between environment and employment, making the emission tax superior to intensity standard.

5 Welfare Analysis

This section performs a welfare analysis of environmental policies. While the previous section uncovers the major mechanisms through which environmental policies affect the environmental quality and the unemployment rate, this section highlights the economic tradeoff of the optimality of environmental policies.

We define welfare as the sum of the total steady-state surplus and the value of the aggregate emission level. The use of total steady-state surplus as a measure of welfare is common in the search and matching literature (Hosios, 1990; Pissarides, 2000). We give this welfare function a novel use by extending it to include the level of pollutant emission. Welfare can be written as

\[
W = (1 - u)(1 - \phi)(p_c A - rk) + \phi(p_d A - C(\bar{x} - x) - rk) - \theta urk - \psi x,
\]

(30)

where \( \psi \geq 0 \) captures the relative social value of each emission level. Welfare is equal to the total flow of net output, which is the number of workers in the clean sector \((1 - u)(1 - \phi)\) multiplied by the net output \(p_c A - rk\), plus the number of workers in the polluting sector \((1 - u)\phi\) multiplied by the net output \(p_d A - C(\bar{x} - x) - rk\), minus the flow cost of job creation \(vrk = \theta urk\) and the social value of emission \(\psi x\).

Efficient allocation \((x, \theta, \phi)\) maximizes welfare (30) subject to equation (10). From the first-order conditions of this maximization problem, it is straightforward to see that generally, the decentralized equilibrium does not optimize the social welfare. Therefore, the equilibrium allocation can be improved. Similar to the two-sector models in Acemoglu (2001), the socially optimal market tightness is standard as in the Hosios condition (Hosios, 1990; Pissarides, 2000). The number of job creation is optimal only if the workers’ bargaining power \(\beta\) is identical to the elasticity of the matching function \(\eta(\theta)\) with respect to market tight. If \(\beta\) is larger than \(\eta(\theta)\), social welfare can be improved by creating more jobs. Since this paper is concerned with the emission and the composition of jobs, we will not discuss this issue in detail.

Using the steady-state level of emission stocks (17), the welfare function can be written as:

\[
W = (1 - u)[(1 - \phi)(p_c A - rk) + \phi(p_d A - C(\bar{x} - x) - \psi x - rk)] - \theta urk.
\]

(31)

Let superscript \(W\) denote the socially optimal object. Differentiating the welfare function
with respect to \( x \), the first-order condition is

\[
\frac{\partial W}{\partial x} = C'(\bar{x} - x^W) - \psi = 0. \quad (32)
\]

This condition is standard in the literature. The socially optimal level of emission per job \( x^W \) equates the marginal social benefit of abatement to its marginal social cost. To abate an additional unit of emission, the society pays the marginal cost of abatement \( C'(\bar{x} - x^W) \). Meanwhile, this additional unit of abatement reduces a unit of the social cost of emission \( \psi \). In the decentralized equilibrium, dirty-good jobs produce without considering the social cost of emission and thus, do not abate. The externality of emission along its intensive margin calls for environmental policies.

Differentiating the welfare function with respect to \( \phi \), the first-order condition is

\[
\frac{\partial W}{\partial \phi} = (1 - u) \left\{ [p_d(\phi^W)A - C(\bar{x} - x) - rk] - [p_c(\phi^W)A - rk] \right\}
\]

\[
- (1 - u)\psi x
\]

\[
+ (1 - u) [\varepsilon_{p_d,\phi}(\phi^W) - \varepsilon_{p_c,1-\phi}(\phi^W)] = 0.
\]

\[
\text{Expected Net Flow Output Gain}
\]

\[
\text{Expected Social Cost of Emission along the Extensive Margin}
\]

\[
\text{Expected Change in Market Values of Existing Firms}
\]

While the cost in creating a vacancy of either type is identical, it makes no difference in the social welfare by creating one type of vacancy or the other. Shifting an additional unit of production from the nonpolluting sector to the polluting sector gains the net flow output of a dirty-good job \( p_d(\phi)A - C(\bar{x} - x) - rk \) at the expense of the net flow output of a clean-good job \( p_c(\phi)A - rk \). In the decentralized equilibrium, the free-entry conditions ensure that the flow profits are identical in the two sectors (i.e., \( p_d(\phi)A - C(\bar{x} - x) = p_c(\phi)A \)). As a result, the expected net flow output gain is zero. Hence, the derivative can be further reduced to

\[
\frac{\partial W}{\partial \phi} = (1 - u) \left[ - \psi x + \varepsilon_{p_d,\phi}(\phi) - \varepsilon_{p_c,1-\phi}(\phi) \right]. \quad (33)
\]

An additional unit of production from the polluting sector generates emission along the extensive margin and costs the society by \( \psi x > 0 \). Of course, firms are not concerned with the social cost of emission when they create dirty-good jobs. For this reason, social welfare improves if the market share of polluting firms decreases. This externality of emission along the extensive margin is another reason for environmental policies.

Shifting the production unit from one sector to the other may change the marginal revenues of all firms. The market value of all existing dirty-good production falls by \( \varepsilon_{p_d,\phi}(\phi) \) because dirty goods become more abundant. Similarly, the market value of all existing
clean-good production rises by $\varepsilon_{p_c,1-\phi}(\phi)$ because clean goods become less abundant. Since marginal revenue and marginal cost does not necessarily equal at the position level in the decentralized equilibrium, the expected change in market values of existing firms is in general nonzero, calling for policies that correct this job allocation externality.\footnote{This kind of job allocation externality exists in Acemoglu (2001) as well.}

Suppose $\varepsilon_{p_d,\phi}(\phi) < \varepsilon_{p_c,1-\phi}(\phi)$. It is beneficial to the society to reduce the market share of the polluting sector because the drop in the market value of existing dirty-good production is larger than the rise in the market value of existing clean-good production. Since the decrease in the market share also reduces the negative externality created by the extensive margin of emission, a social planner should reduce the market share. However, if $\varepsilon_{p_d,\phi}(\phi) > \varepsilon_{p_c,1-\phi}(\phi)$, whether a social planner should increase or decrease the market share of the polluting sector depends on the relative size of the social cost of emission along the extensive margin and the change in the market values of existing firms. Environmental policies that purpose to solely reduce emission along both margins may worsen social welfare.

### 5.1 The Optimality of Environmental Policies

Next, we investigate the optimality of simple emission tax, intensity standard, and revenue-neutral emission tax. This investigation is informative for two reasons. First, it is relevant to policymakers in selecting among these policies. Second, these investigations complete the analysis of the characterization of socially optimal environmental policies.

From equation (32), a social planner could impose a Pigouvian tax on emission so that simple emission tax policy $\mathcal{E}T(\psi)$ internalizes the externality of emission along its intensive margin. Given the Pigouvian tax rate $\psi$, the marginal social benefit of abatement $\psi$ equals the marginal social cost of abatement $C'(\bar{x} - x^{\mathcal{E}T}(\psi))$. This condition does not account for the tax-interaction effect (Bovenberg and Goulder, 1996) because this paper abstracts pre-existing payroll tax from the model. By incorporating the pre-existing tax, the optimal amount of emission along its intensive margin will account for the tax-interaction effect, and the intuition will be the same as in the literature. The flow cost of $\mathcal{E}T(\psi)$ equals $T^{\mathcal{E}T}(\psi) = C(\bar{x} - x^{\mathcal{E}T}(\psi)) + \psi x^{\mathcal{E}T}(\psi)$. Hence, the welfare function becomes:

\[
W^{\mathcal{E}T}(\psi) = (1 - u) \left[ (1 - \phi)(p_c A - r k) + \phi(p_d A - C(\bar{x} - x^{\mathcal{E}T}(\psi)) - \psi x^{\mathcal{E}T}(\psi) - r k) \right] - \theta urk. \tag{34}
\]

Notice that each dollar increase in the emission tax payment increases one dollar of the government revenue. Therefore, the term $\psi x^{\mathcal{E}T}(\psi)$ does not reflect the transfer of the emission tax payment. Instead, it captures the welfare loss from the emission stocks $\psi \chi$ in the welfare equation (31), which is identical to the emission tax payment. The derivative of
the welfare with respect to \( \phi \) evaluated at \( \phi^{ET}(\psi) \) can be expressed as follows:

\[
\frac{\partial W^{ET}(\psi)}{\partial \phi} \bigg|_{\phi=\phi^{ET}(\psi)} = (1-u) \left\{ \left[ \psi_{d}(\phi^{ET}(\psi)) A - C(\bar{x} - x^{ET}(\psi)) - r k \right] - \left[ \psi_{c}(\phi^{ET}(\psi)) A - r k \right] \right\}
\]

Expected Net Flow Output Gain
\[
-(1-u)\psi_{c}^{ET}(\psi)
\]

Expected Social Cost of Emission along the Extensive Margin
\[
+(1-u)(\psi_{d,\phi}(\phi^{ET}(\psi)) - \psi_{c,1-\phi}(\phi^{ET}(\psi)))
\]

Expected Change in Market Values of Existing Firms

While a social planner considers the change in the net flow output, firms consider their flow profit \( p_{d}(\phi^{ET}(\psi)) A - T^{ET}(\psi) \) in determining whether to create jobs. Nevertheless, a social planner consider the emission tax payment as transfer from firms to the government, not as welfare gain or loss. Free-entry conditions ensure the flow profits in the two sectors are identical under simple emission tax; hence, shifting the production from the nonpolluting sector to the polluting sector does not create additional profits. That is, \( p_{d}(\phi^{ET}(\psi)) A - T^{ET}(\psi) = p_{c}(\phi^{ET}(\psi)) A \). Hence, the net flow output gains by \( \psi x^{ET}(\psi) \) if shifting a unit of production from the nonpolluting sector to the polluting sector. In other words, creating more jobs in the polluting sector improves the social welfare under simple emission tax \( ET(\psi) \) if only the expected net flow output is considered. This gain is offset by the social cost of emission along the extensive margin. Generally, the expected change in market values of existing firms is nonzero. Therefore, the derivative can be further reduced to

\[
\frac{\partial W^{ET}(\psi)}{\partial \phi} \bigg|_{\phi=\phi^{ET}(\psi)} = (1-u) \left[ \psi_{d,\phi}(\phi^{ET}(\psi)) - \psi_{c,1-\phi}(\phi^{ET}(\psi)) \right].
\]

\( \psi^{ET}(\psi) \) does correct the emission externality along both margins and the job allocation externality along the expected net flow output gain. Simple emission tax in general fails to internalize the job allocation externality because firms do not account for the impact of creating vacancies on the market values of existing firms.

Next, we analyze the optimality of intensity standard \( IS(x^{ET}(\psi)) \) that internalizes the externality of emission along the intensive margin (i.e., the intensity standard with an implicit emission tax rate \( \psi \)). Hence, the socially optimal standard \( \hat{x} \) equals \( x^{ET}(\psi) \), which equalizes the marginal social cost of abatement to its marginal social benefit.

Under \( IS(x^{ET}(\psi)) \), free-entry conditions again equalize the flow profits in the two sectors. That is, \( p_{d}(\phi^{IS}(\psi)) A - C(\bar{x} - x^{ET}(\psi)) = p_{d}(\phi^{IS}) A \). Hence, there is no expected net flow output gain because of the shift of a unit of production from a nonpolluting sector to a polluting sector. Hence, we have

\[
\frac{\partial W^{IS}(x^{ET}(\psi))}{\partial \phi} \bigg|_{\phi=\phi^{IS}(x^{ET}(\psi))} = (1-u) \left[ -\psi x^{ET}(\psi) + \psi_{d,\phi}(\phi^{IS}(x^{ET}(\psi))) - \psi_{c,1-\phi}(\phi^{IS}(x^{ET}(\psi))) \right].
\]
While $L S(x^{ET}(\psi))$ internalizes the expected social cost of emission along the intensive margin, it fails to correct the externality arising from the extensive margin of emission. Similar to simple emission tax, intensity standard fails to internalize the job allocation externality because firms do not account for the impact of creating vacancies on the market values of existing firms.

Next, we investigate the welfare of the revenue-neutral emission tax policy $R N(\psi, h)$, in which a social planner imposes an emission tax $\psi$ per unit of $x$ so that the intensive margin of emission is socially optimal. This policy rebates $h_d$ evenly to each dirty-good job and even redistributes the rest of the emission tax revenue to each clean-good job. The total emission tax revenue equals $\psi x^W \phi (1 - u)$; therefore, $h_d \in [0, \psi x^W]$. A revenue-neutral emission tax policy requires the rest of the tax revenue is even redistributed to each clean-good job, equal to $h_c = (\psi x^W - h_d) \phi / (1 - \phi)$.

Similar to emission tax, the expected net flow output gain is zero. The transfers, including emission tax payments and rebates, do not bring welfare gain or loss to the society. However, in determining whether to create jobs in either sector, these transfers are considered in calculating profits, and zero-profit conditions equate the net flow profits in the two sectors. The expected net flow output gain equals $(1 - u)(\psi x^W + h_d - h_c)$. Since the emission tax payment equals the social cost of emission along the extensive margin, the derivative can be reduced to

$$\frac{\partial W^{RN}(\psi, h)}{\partial \phi} \bigg|_{\phi = \phi^{RN}(\psi, h)} = (1 - u) \left[ h_d - h_c + \varepsilon_{pd, \phi}(\phi^{RN}(\psi, h)) - \varepsilon_{pc,1-\phi}(\phi^{RN}(\psi, h)) \right].$$

To attain the social optimum, the rebate differential should be set to $\varepsilon_{pd, \phi}(\phi^{RN}(\psi, h)) - \varepsilon_{pc,1-\phi}(\phi^{RN}(\psi, h))$ as follows:

$$h_d - \frac{\psi x^W \phi^{RN}(\psi, h) - h_d \phi^{RN}(\psi, h)}{1 - \phi^{RN}(\psi, h)} = \varepsilon_{pd, \phi}(\phi^{RN}(\psi, h)) - \varepsilon_{pc,1-\phi}(\phi^{RN}(\psi, h)).$$

The interior solution of $h_d$ is given by $\psi x^W \phi^{RN} + (1 - \phi^{RN})[\varepsilon_{pd, \phi}(\phi^{RN}) - \varepsilon_{pc,1-\phi}(\phi^{RN})]$ and the corresponding rebate to each clean-good job is given by $\psi x^W \phi^{RN} - \phi^{RN} \varepsilon_{pd, \phi}(\phi^{RN}) - \varepsilon_{pc,1-\phi}(\phi^{RN})$. The rebate of the tax revenue depends largely on the elasticity of revenue. Suppose $\varepsilon_{pd, \phi}(\phi^{RN}) - \varepsilon_{pc,1-\phi}(\phi^{RN}) < 0$. That is, the elasticity of revenue is larger in the polluting sector. This economy is better off shrinking the market share of the polluting sector because the increase in the total revenue in the polluting sector outweighs the decrease in the total revenue in the other sector. This explains why a social planner will rebate less to each dirty-good job if $\varepsilon_{pd, \phi}(\phi^{RN}) - \varepsilon_{pc,1-\phi}(\phi^{RN}) < 0$. The rebates should be evenly distributed regardless of sector only if $\varepsilon_{pd, \phi}(\phi^{RN}) = \varepsilon_{pc,1-\phi}(\phi^{RN})$.

Given the Pigouvian tax rate, to maximize social welfare, the general solution of the
Intuitively, $\varepsilon_{pd,\phi} - \varepsilon_{pc,1-\phi} < -\psi x W \phi^{RN} / (1 - \phi^{RN})$ implies that the revenue elasticity in the polluting sector is so large that it is beneficial to shrink the market share of the polluting sector. All the emission tax revenues are rebated to the polluting sector, leaving no rebate to the other sector. After rebating the tax revenue, there is still too much the market share of the polluting sector, and the job composition is not socially optimal. A similar intuition can be applied to the other extreme in which $\varepsilon_{pd,\phi} - \varepsilon_{pc,1-\phi} > -\psi x W \phi^{RN} / (1 - \phi^{RN})$. Only if $-\frac{\psi x W}{1 - \phi} \leq \varepsilon_{pd,\phi} - \varepsilon_{pc,1-\phi} \leq -\psi x W$, the emission per job and the market share are socially optimal. The following proposition summarizes the result.

**Proposition 8.** For all $\psi \in \mathbb{R}^+$, if $-\frac{\psi x W \phi^{RN}(\psi, h)}{1 - \phi^{RN}(\psi, h)} \leq \varepsilon_{pd,\phi} - \varepsilon_{pc,1-\phi} \leq -\psi x W$, $x^{RN}(\psi, h)$ and $\phi^{RN}(\psi, h)$ are socially optimal, where $h_d$ and $h_c$ are given by equations (35) and (36).

Under a certain condition, this policy reaps a triple dividend. First, its Pigouvian tax on emission internalizes the externality of emission along the intensive margin. Second, the rebates of the tax revenue correct the externality along the external margin. Third, the rebates fine-tune the job composition, the distortion of the labor market.

By design, the intensive margin of emission is socially optimal under all the three policies, $\mathcal{R}N(\psi, h)$, $\mathcal{ET}(\psi)$, and $\mathcal{IS}(x^{\mathcal{ET}}(\psi))$. However, the job composition is unlikely socially optimal under $\mathcal{ET}(\psi)$ or $\mathcal{IS}(x^{\mathcal{ET}}(\psi))$. Under $\mathcal{R}N(\psi, h)$, the emission tax revenue can be rebated to correct the externality arising from the extensive margin of emission and to guarantee the job composition at its optimum under a relatively weak assumption. Recall from Proposition 7 that if $\varepsilon_{pd,\phi} \leq \varepsilon_{pc,1-\phi}$ in the absence of environmental policy, there always exists an emission tax rate $\tau > 0$ such that $\mathcal{R}N(\tau, h)$ improves environment and depresses unemployment. This paper contributes to the literature by highlighting the importance of the revenue elasticities in these two sectors when the tradeoff between environment and employment and the social optimality of environmental policies are studied. Indeed, Propositions 7 and 8 imply that if $\varepsilon_{pd,\phi} \leq \varepsilon_{pc,1-\phi}$ in the absence of environmental policy, the introduction of $\mathcal{R}N(\tau, h)$ with a mild emission tax rate $\tau$ will have no significant impact on the two revenue elasticities, and thus the corresponding condition in
Proposition 8 is likely satisfied. In other words, once $\varepsilon_{p_d, \phi} \leq \varepsilon_{p_c, 1-\phi}$ is satisfied, the introduction of $\mathcal{RN}(\tau, h)$ with a mild $\tau$ is, though not necessarily socially optimal, likely welfare-improving and could avoid the tradeoff between environment and employment.

6 Conclusion

This paper constructs a two-sector search equilibrium model to study the tradeoff between environmental quality and employment. We show that the introduction of the class of policies that only impose a higher flow cost on the polluting sector without providing any incentive to increase emission from the intensive margin trade the environmental quality off against the employment opportunity.

To break the inevitability of the tradeoff requires a policy to affect the flow cost of production in both the polluting and the nonpolluting sectors. A revenue-neutral emission tax with equal rebates to both sectors and a proper tax rate is shown to improve both employment and environmental outcomes under the condition that the revenue elasticity in the polluting sector is larger than that in the nonpolluting in absolute value. We show that on the grounds of employment and the environmental quality, there always exist intensity standards that dominate any emission tax if the tax revenue is not considered, and for any intensity standard, there exists no emission tax that is superior. If the revenue-neutral emission tax improves environmental outcomes without increasing unemployment, this tax policy dominates any simple emission tax and a large family of intensity standards.

Concerning welfare, a Pigouvian tax corrects the environmental externality regarding the emission from each polluting firm but not the job allocation externality in the labor market. Thus, a Pigouvian tax cannot achieve the socially optimal allocation of production between sectors. To internalize the job allocation externality, the tax revenue could be rebated to both sectors such that the rebate differential equals the revenue elasticity differential. Under certain conditions on the revenue elasticity differential, the revenue-neutral Pigouvian tax with rebates can achieve socially optimal allocation of production between sectors.

This paper opens up many potential directions of extensions for future research. First, it would be interesting to explore whether the emission tax revenue can be rebated properly to resolve other externalities in the frictional labor market. For example, we can incorporate the entry/exit externality of unemployed workers by adding an out-of-labor-force status for workers. Second, we can model worker heterogeneity in human capital and firm heterogeneity in productivity to analyze the effects of environmental policies on a wage distribution. Third, we can introduce dynamic stochasticity of productivity to incorporate real business cycle analysis and compare the political feasibility of environmental policies in recessions and booms. Finally, it would be useful to test some of the predictions of this model.


7 Appendix: Proof

7.1 Proof of Proposition 1

We show that a unique $\phi^*$ exists. Since $\lim_{\phi \to 0} \Lambda(\phi) = \infty$, $\lim_{\phi \to 1} \Lambda(\phi) = \infty$, and $\Lambda(\phi)$ is continuous on $(0, 1)$, there must exist at least one $\phi^* \in (0, 1)$ such that $\Lambda(\phi^*) = 0$ from revenue differential (16). Since $\Lambda'(\phi) < 0$ and the R.H.S. is independent of $\phi$, the $\phi^* \in (0, 1)$ is unique. Given the unique $\phi^*$, the unique $p_d(\phi^*)$ and $p_c(\phi^*)$ are pinned down by equation (11).

Lastly, we show that a unique $\theta^*$ exists. When $\theta$ tends to zero, the R.H.S. of equation (15) approaches positive infinity because of $\lim_{\theta \to 0} q(\theta) = \infty$. When $\theta$ tends to positive infinity, its R.H.S. approaches negative infinity because of $\lim_{\theta \to \infty} \theta q(\theta) = 0$ and $\lim_{\theta \to \infty} \theta^2 q(\theta) = \infty$. As the L.H.S. is a positive constant, the intermediate value theorem ensures that there exists at least one $\theta^* > 0$ such that equation (15) is satisfied. Since the R.H.S. strictly decreases with $\theta$, $\theta^*$ is unique. Substituting the unique $\theta^*$ into equation (10) yields the unique steady state $u^*$ because $u$ strictly decreases with $\theta$. Given that $x^*$, $\phi^*$, $p_d(\phi^*)$, $p_c(\phi^*)$, $\theta^*$, and $u^*$ are unique, it is straightforward to complete the rest of the proof.

7.2 Proof of Proposition 6

First, $\varphi_{RN}(\tau, h) = \varphi_{ET}(\tau)$ for all $\tau > 0$. $\varphi_{ET}(\tau)$ is lower than the emission level in the absence of environmental policies, where the last inequality follows from Lemma 1. To prove Proposition 6, it is sufficient to show that $\varepsilon_{p_d, \phi} \leq \varepsilon_{p_c, 1-\phi}$ implies $\frac{d u_{RN}^*(\tau, h_c)}{d \tau} \leq 0$.

Next, we define

$$
\Pi_{RN}^c \equiv p_c(\phi_{RN}^*(\tau, h_c))A - rk - z - \frac{\beta \theta_{RN}^*(\tau, h_c)}{1+\beta} r k + \tau x_{RN}^*(\tau, h_c) \phi_{RN}^*(\tau, h_c).
$$

According to the steady-state unemployment (10) and the zero-profit condition in the non-polluting sector (28), we can claim that

$$
\frac{d \Pi_{RN}^c}{d \tau} \geq 0 \text{ iff } \frac{d u_{RN}^*(\tau, h)}{d \tau} \leq 0.
$$

Suppose $d \Pi_{RN}^c/d \tau > 0$. The zero-profit condition in the nonpolluting sector (28) ensures that $d \theta_{RN}^c/d \tau > 0$. Using the steady-state unemployment rate (10), it is straightforward to show that $du_{RN}^*(\tau, h)/d \tau < 0$. Similar arguments can be applied to establish the claim.

Totally differentiating $\Pi_{RN}^c$ with respect to $\tau$, we have

$$
\frac{d \Pi_{RN}^c}{d \tau} = \frac{dp_c(\phi_{RN}^*)A}{d \tau} + \frac{d \theta_{RN}^*}{d \tau} \frac{r k + x_{RN}^* \phi_{RN}^* + \tau x_{RN}^* d x_{RN}^*}{1+\beta} + \tau x_{RN}^* \frac{d \phi_{RN}^*}{d \tau}.
$$

30
Note that \( \lim_{\tau \to 0} \frac{dx_{RN}}{d\tau} \) < \( \infty \) and \( \lim_{\tau \to 0} \frac{d\phi_{RN}}{d\tau} \) < \( \infty \). Hence, we have

\[
\lim_{\tau \to 0} \frac{d\Pi_c^{RN}}{d\tau} = \lim_{\tau \to 0} \frac{dp_c(\phi_{RN})A}{d\tau} - \lim_{\tau \to 0} \frac{d\phi_{RN}}{d\tau} \frac{\beta}{1 + \beta^r} k + \lim_{\tau \to 0} x_{RN} \phi_{RN}.
\]

Next, we claim that

\[
\lim_{\tau \to 0} \frac{d\Pi_c^{RN}}{d\tau} \geq 0 \iff \lim_{\tau \to 0} \frac{dp_c(\phi_{RN})A}{d\tau} + \lim_{\tau \to 0} x_{RN} \phi_{RN} \geq 0.
\]

Suppose \( \lim_{\tau \to 0} \frac{d\Pi_c^{RN}}{d\tau} > 0 \). If the claim is invalid, then \( \lim_{\tau \to 0} \frac{dp_c(\phi_{RN})A}{d\tau} + \lim_{\tau \to 0} x_{RN} \phi_{RN} \leq 0 \). Using equation (29), we can show that \( \lim_{\tau \to 0} \frac{d\Pi_c^{RN}}{d\tau} > 0 \) implies \( \lim_{\tau \to 0} \frac{dp_c(\phi_{RN})A}{d\tau} + \lim_{\tau \to 0} x_{RN} \phi_{RN} > 0 \). Hence, we have

\[
\lim_{\tau \to 0} \frac{d\Pi_c^{RN}}{d\tau} = \lim_{\tau \to 0} \frac{dp_c(\phi_{RN})A}{d\tau} + \lim_{\tau \to 0} x_{RN} \phi_{RN} - \lim_{\tau \to 0} \frac{d\phi_{RN}}{d\tau} \frac{\beta}{1 + \beta^r} k < 0.
\]

A contradiction arises. Hence, \( \lim_{\tau \to 0} \frac{d\Pi_c^{RN}}{d\tau} > 0 \) implies \( \lim_{\tau \to 0} \frac{dp_c(\phi_{RN})A}{d\tau} + \lim_{\tau \to 0} x_{RN} \phi_{RN} > 0 \). Similar arguments can be applied to establish the claim. Following the two claims in this proof,

\[
\lim_{\tau \to 0} \frac{dp_c(\phi_{RN})A}{d\tau} + \lim_{\tau \to 0} x_{RN} \phi_{RN} \geq 0 \iff \lim_{\tau \to 0} \frac{d\phi_{RN}}{d\tau} \leq 0.
\]

Next, we derive the sufficient and necessary condition to have \( \lim_{\tau \to 0} \frac{dp_c(\phi_{RN})A}{d\tau} + \lim_{\tau \to 0} x_{RN} \phi_{RN} > 0 \). Since \( x_{RN} = x^{ET} \) and \( \phi_{RN} = \phi^{ET} \),

\[
\lim_{\tau \to 0} \frac{dp_c(\phi_{RN})A}{d\tau} + \lim_{\tau \to 0} x_{RN} \phi_{RN} > 0 \implies \lim_{\tau \to 0} \frac{dp_c(\phi^{ET})A}{d\phi^{ET}} d\phi^{ET} + \lim_{\tau \to 0} x^{ET} \phi^{ET} > 0.
\]

Under simple emission tax, the revenue differential is given by \( \Lambda(\phi^{ET}(\tau)) = C(\bar{x} - x^{ET}(\tau)) + \tau x^{ET}(\tau) \). Totally differentiating the revenue differential with respect to \( \tau \), the envelop theorem implies that \( d\phi^{ET}/d\tau = x^{ET}[\Lambda'(\phi^{ET})]^{-1} \). The above inequality can therefore be simplified as follows:

\[
\lim_{\tau \to 0} \frac{dp_c(\phi^{ET})}{d\phi^{ET}} \left( \frac{dp_d(\phi^{ET})}{d\phi^{ET}} - \frac{dp_c(\phi^{ET})}{d\phi^{ET}} \right)^{-1} + \phi^{ET} > 0
\]

\[
\lim_{\tau \to 0} \varepsilon_{p_d,\phi}(\phi^{ET}) < \lim_{\tau \to 0} \varepsilon_{p_c,1-\phi}(\phi^{ET}),
\]

where \( \varepsilon_{p_d,\phi}(\phi^{ET}) \equiv \phi^{ET} \frac{dp_d(\phi^{ET})A}{d\phi^{ET}} \) and \( \varepsilon_{p_c,1-\phi}(\phi^{ET}) \equiv (1 - \phi^{ET}) \frac{dp_c(\phi^{ET})A}{d(1-\phi^{ET})} \). We used \( \frac{dp_d(\phi^{ET})}{d\phi^{ET}} - \frac{dp_c(\phi^{ET})}{d\phi^{ET}} < 0 \). If \( \tau \) tends to zero, \( \phi^{ET}(\tau) \) approaches the \( \phi \) in the absence of
environmental policy. A similar argument can be applied to show that $\varepsilon_{p,\phi} \leq \varepsilon_{p,1-\phi}$ in the absence of environmental policy implies that $\frac{d\nu_{RN}(\tau,h)}{d\tau} \leq 0$.

References


